

February 7, 2008

UCRHEP-T121

Electroweak Effective Lagrangians

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ABSTRACT

In this paper I review several aspects of the use of effective lagrangians in (mainly) electroweak physics. The conditions under which this approach is reliable and useful, as well as the limitations of the formalism are detailed. Various applications are also presented.

1. Introduction.

When constructing models aimed at describing physics behind a certain set of phenomena the resulting formalism is understood to have, except in the most ambitious cases (such as superstrings) a limited range of validity. For example, hydrodynamics is very successful in describing liquid motion at scales much larger than the typical atomic size. Below such scale the model describing the behaviour of this system changes dramatically as the dynamical variables are different. Of course, barring technical difficulties, the hydrodynamic description of a fluid can be obtained from the microscopic one by defining the appropriate macroscopic observables and deducing the quantum-average equations of motion. Going a step further one can imagine deducing from the quantum theory the $O(\hbar)$ corrections to the Navier-Stokes equation. Then we would find that, as we consider smaller and smaller distances, these corrections will become increasingly important, to the point that when we reach distances comparable to the atomic scale the whole series in \hbar must be included in order to describe the system.

In the realm of high energy physics there is a direct parallel with the above observations. When the description of a certain set of phenomena in terms of a model is desired, the first ingredient in the construction of the model is the determination of the relevant particles, corresponding to the dynamical variables of the theory. Then, often based on experimental evidence, the symmetries obeyed by the model are specified. Finally the most general (local) lagrangian is constructed containing the fields corresponding to the above particles obeying the said symmetries is constructed.

To determine the physical content of an effective theory it is necessary to understand how it is generated (for a review see Ref. 1). Suppose we have a model from which we wish to derive a description of phenomena at energies below a certain scale Λ . This description is formally obtained from an effective action S_{eff}^Λ whose 1PI functions are generated by loops containing only excitations of energy $\geq \Lambda$; equivalently S_{eff}^Λ is derived from a functional integral by integrating

over all Fourier components of energy $\geq \Lambda$. The resulting expression will be a non-local function of the low energy fields (*i.e.*, those whose Fourier components correspond to energies $< \Lambda$). Note that a simple description of this effective action might require the low energy fields to be non-linear functions of the original fields. This is the case, for example, in QCD where the low energy physics is best described in terms of composite quark operators corresponding to the various mesons and baryons.

The effective action can be expanded in an infinite series of local (effective) operators \mathcal{O}_i containing only low energy fields, with Λ -dependent coefficients^{1,2}

$$S_{\text{eff}}^\Lambda = \int d^4x \mathcal{L}_{\text{eff}} = \int d^4x \sum_i \alpha_i(\Lambda) \mathcal{O}_i. \quad (1.1)$$

which defines the effective lagrangian \mathcal{L}_{eff} . The effective lagrangian comprises all virtual heavy physics effects; as long as we remain within its realm of applicability (at scales below Λ), it provides a unitary, consistent and complete description of all low energy phenomena;³ in particular all the Green functions of the heavy theory at low energies can be derived from (1.1).

The *form* of the effective lagrangian in (1.1) is independent of the model from which it is derived. It then follows that one can parametrize all possible heavy physics effects by an expression of this type, where, as mentioned above, the \mathcal{O}_i are only constrained by the symmetries of the low energy physics. This expression for \mathcal{L}_{eff} will provide a model-independent, consistent, complete and unitary description of heavy physics effects which respects all low energy constraints required by the corresponding symmetries; for example, Green functions including heavy physics effects are parametrized in this manner independently of the details of the heavy physics. It is because of its generality that the effective lagrangian approach is ideally suited for studying possible effects of physics beyond the Standard Model.

The effective lagrangian contains several length scales; some of these are identified with the properties of the particles under consideration, such as the corresponding Compton wavelengths.

Other scales are not so associated, and are related to the physics underlying this theory. The model also possesses a number of coupling constants that are determined using experimental data. It is expected (on æsthetic grounds) that a fundamental theory will have a very small number of parameters; but a more modest model, aiming at understanding a limited set of processes, can have a large number of such couplings, in some cases an infinite number. These models have less predictability than an ideal fundamental theory, but, even in the case where there are an infinite number of couplings, the models do have predictive power, as we will see below.

I have not mentioned renormalizability in connection with the above models since they are not supposed to be a good description of nature above a certain energy scale, and so ultra-violet divergences are absent. Since renormalizability is not imposed, the lagrangians will in general contain an infinite set of local operators constructed out of the dynamical variables of the theory and satisfying the symmetries of the model. Each such operator will be multiplied by an undetermined coupling constant (dimensionless or dimensionful) whose precise value is determined by the unknown heavy physics. These constants parametrize all heavy physics effects at low energies.

The presence of an infinite set of couplings leads, via “standard arguments”, to a complete lack of predictability, which apparently implies that this type of model has little practical interest. This is in fact not so: we will see that in these models one can define a hierarchy in the operators present in the lagrangian; to any order in this hierarchy there is a finite set of couplings to consider, and so the model can produce non-trivial predictions. Within the range of applicability of the model the contributions to any process is dominated by the lowest-order operators. One can also estimate the corrections to these predictions produced by the higher order operators; these models have predictive power and also provide information about their range of applicability.

In the following I will use the label “effective theories” to denote the set of models described above. I will show that, despite their not being “fundamental”, they are extremely useful in

understanding physics in a limited range of scales. It is the purpose of this paper to present a review of this type of models, to describe their properties and advertise their usefulness. Many of the ideas presented in this review have appeared elsewhere.^{3,1,4,5,6,7} One purpose of this review is to present these ideas and summarize the conclusions that have been drawn from them.

The contents of this paper are the following. In the next section the symmetries local and global of effective models are discussed, with special attention paid to the issue of gauge invariance. Section 3 presents the construction of the effective lagrangian for electroweak interactions, while section 4 describes the determination of the order of magnitude of the (unknown) coefficients which appear in it. Section 5 discusses the use of the equations of motion to reduce the number of terms in the effective lagrangian; section 6 presents a brief description of the problems associated with unitarity within the context of effective theories. Section 7 illustrates the use of effective lagrangians in calculating radiative corrections. Applications of the effective lagrangian parametrization are presented in section 8 where the limits on various coefficients derived from known data are summarized and, also, the expected sensitivity of future colliders to the new interactions is presented. Conclusions are given in section 9 and several useful expressions and a simple calculation are presented in the appendices.

The notation used in this paper when referring to the Standard Model fields follows that of [40]; it is summarized here for convenience. The fields are

$$\begin{aligned}
W_\mu^I &: SU(2)_L \text{ gauge field;} \\
B_\mu &: U(1)_Y \text{ gauge field;} \\
G_\mu^A &: SU(3)_c \text{ gauge field;} \\
\phi &: \text{scalar doublet;} \\
\ell_l &: \text{left-handed lepton doublet with charged lepton } l, \ l = e, \mu, \tau; \\
l_R &: \text{right-handed charged lepton, } l = e, \mu, \tau; \\
q &: \text{left-handed quark doublet;} \\
u_R, \ d_R &: \text{right-handed quark fields, up and down type respectively;}
\end{aligned} \tag{1.2}$$

The gauge field curvatures are

$$\begin{aligned}
W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g\epsilon_{IJK}W_\mu^J W_\nu^K; \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu; \\
G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_s f_{ABC}G_\mu^B G_\nu^C.
\end{aligned} \tag{1.3}$$

The corresponding gauge couplings are g for $SU(2)$, g' for $U(1)$ and g_s for $SU(3)$. The structure constants for $SU(3)$ are denoted by f_{ABC} .

The covariant derivative for a colorless $SU(2)$ doublet of hypercharge Y is

$$D_\mu = \partial_\mu - ig\frac{1}{2}\sigma_I W_\mu^I - ig'Y B_\mu. \tag{1.4}$$

The hypercharge assignments are $Y(\phi) = 1/2$, $Y(\ell_l) = -1/2$, $Y(l_R) = -1$, $Y(q) = 1/6$, $Y(u_R) = 2/3$, $Y(d_R) = -1/3$. I will also need the matrix $\epsilon = i\sigma_2$ and the field $\tilde{\phi} = \epsilon\phi^*$.

There are several topics which will not be discussed in this review. I will not cover the BESS model,⁸ for which I refer the reader to the literature. No mention will be made of the application of effective lagrangians to low energy strong interactions. This has been covered in many excellent publications (see Ref. 9; for a recent review see Ref. 10). On the weak interactions area there

are many publications dealing with specific models and their low energy phenomenology (see for example the extensive review in Ref. 11 and references therein), this approach will not be followed since, by definition, is model specific.

Several authors^{12,13,14} have also considered a mixed approach where the effective models are studied beyond their UV cutoff; this generates, among other problems, violations of tree level unitarity.^a To fix this problem several methods of “unitarizing” the models are used (see the above references as well as Ref. 15; for a recent review see Ref. 16). This approach involves specific assumptions regarding the new physics and to this extent runs counter to the philosophy of using an effective lagrangian parametrization. This approach will not be presented here. I refer the reader to the above references for a thorough discussion.

There are also several publications where the coefficients of the effective lagrangian terms are taken to be form factors.¹⁷ This will also not be covered in this review since it is hard to translate the results obtained using the form-factor approach to the language used in this paper. I refer the interested reader to the literature.

2. Symmetries.

One of the most important ingredients in the construction of effective lagrangians, denoted by \mathcal{L}_{eff} , is the determination of the symmetries respected by the models.

2.1. *Global symmetries.*

Global symmetries such as those associated with lepton or baryon number conservation, are imposed based on experimental evidence. The requirement that \mathcal{L}_{eff} satisfies them just imposes an added restriction which reduces the number of allowed operators. Should experiments demonstrate a violation of these symmetries, one can always introduce effective interactions which

^a See section 6 for a discussion on this point

violate them. These interactions will then be associated with the scale at which violations of the global symmetry are generated. For very small deviations (as, for example, for baryon number violation) such a scale will be very large compared to the typical scale of the low energy physics. Thus, for example, there is no fundamental theoretical principle which forbids the introduction, in an electroweak effective lagrangian, of the term

$$\frac{1}{\Lambda} \bar{e}_R \sigma_{\mu\nu} \mu_R B^{\mu\nu} \quad (2.1)$$

(the notation is given in Eqs. (1.2)-(1.4) and the comments following them.). This interaction generates the decay $\mu \rightarrow e\gamma$ with a width $\propto 1/\Lambda^2$. The fact that this process has not been seen merely implies that Λ is very large.

2.2. *Local symmetries.*

Many phenomenological models considered in the literature which describe physics beyond the Standard Model effects are not gauge invariant¹⁸, and because of this have been strongly criticized.⁶ It was then pointed out^{19,20} that *any* effective lagrangian can be understood as the unitary gauge limit of a gauge invariant \mathcal{L}_{eff} . In this section I will discuss these issues. Other related considerations concerning gauge invariance and radiative corrections will be presented in section 7.3.

The process by which a lagrangian is made gauge invariant is based on a trick originally invented by Stückelberg (see Refs. 21 and 22). The idea is the following: suppose we have a lagrangian \mathcal{L} depending on some (real) vector fields W_μ^n , $n = 1, 2, \dots, N$, together with some other fields, which I'll denote by χ . Choose then a (Lie) group \mathcal{G} of dimension $D \geq N$, and add $D - N$ auxiliary (real) vector fields \bar{W}_μ^n , $n = N + 1, \dots, D$ assumed to be non-interacting (if desired, the masses for these fields can be taken $\geq \Lambda$); this modification of the effective lagrangian does not affect any low energy observable. Henceforth I will drop the over-bar in these extra fields; the lagrangian (with the new fields) will be denoted by $\mathcal{L}(W; \chi)$.

Using the above vector fields, and the (antihermitian) generators of \mathcal{G} , denoted by $\{T^n\}$ (in any irreducible representation), normalized such that $\text{tr}\{T^n T^m\} = -\delta_{nm}$, I can construct the derivative operator

$$D_\mu = \partial_\mu + \sum_{n=1}^D T^n W_\mu^n. \quad (2.2)$$

Next I introduce a unitary auxiliary field U which, under an infinitesimal \mathcal{G} transformation, behaves as $\delta U = \sum_n \epsilon_n T^n \cdot U$. Finally I define the objects

$$\mathcal{W}_\mu^n = -\text{tr} (T^n U^\dagger D_\mu U). \quad (2.3)$$

With these definitions I now assume that the W^n are in fact gauge fields with all the corresponding properties. Then the \mathcal{W}^n are gauge invariant objects which satisfy

$$\mathcal{W}_\mu^n|_{U=1} = W_\mu^n. \quad (2.4)$$

It follows that

$$\mathcal{L}(W; \chi) = \mathcal{L}(\mathcal{W}; \chi)|_{U=1}, \quad (2.5)$$

which means that the original lagrangian is the unitary gauge limit of a gauge invariant theory. The fields χ are assumed to be gauge invariant.

The price paid in rendering a model gauge invariant is, once \mathcal{G} is chosen, the (possible) introduction of the extra vector fields \bar{W} and the auxiliary field U . Note that even if no extra vector fields are introduced, there is always a group of dimension N : $U(1)^N$. Therefore the above procedure can also be carried out with the original vector fields as gauge fields, though this restricts the allowed choices for \mathcal{G} .

On the basis of the above construction it has been argued²⁰ that gauge invariance has no content. After all, any theory can be considered as a gauge invariant theory, albeit in the unitary

gauge. I disagree with this conclusion. Obvious drawbacks of the procedure on which this statement is based are the arbitrariness in the choice of \mathcal{G} , and the fact that the construction in general disallows a linear realization of the gauge symmetry^b (see section 3.2 for further discussion).

Consider for example the Standard Model in the unitary gauge. Taking the corresponding lagrangian as the terms of dimension ≤ 4 of an effective theory, the above trick can be used to render it an $SU(2) \times U(1)$ *or* a $U(1)^4$ gauge theory. In the latter case experimental results would suggest a number of relations among the coefficients of the operators allowed by the symmetry, such relations would have no fundamental justification. If on the other hand we assume that the above group is $SU(2) \times U(1)$ these relations between the coefficients would be *predicted* by this model, considerably adding to its credibility.

More important is the fact that the non-gauge fields χ are assumed to be gauge singlets. Thus, for example, the relationships between the Z , W and photon couplings to the fermions would, in this approach, be a mere accident (it is certainly possible to impose the Standard Model relations among these couplings, but there would be no justification for this choice). If, on the other hand, the Standard Model local symmetries and the representations carried by the fields are chosen, then these relations are again a success of the model.

These arguments support the claim that gauge invariance does have non-trivial content, not in the abstract sense (since indeed any model can be made gauge invariant under, in general, many gauge groups), but in the practical sense: when it is stated that a given theory is gauge invariant with group \mathcal{G} and the corresponding transformation properties of the fields are specified, the structure of the model, together with the number of unknown parameters, is largely fixed. This *can* be tested by experiment. If gauge invariance had no content these conditions would be naturally predicted irrespective of \mathcal{G} .

These considerations are of importance when doing loop calculations: the sensible approach

^b The exception corresponds to the case where \mathcal{G} is a product of $U(1)$ factors.

is then to choose a renormalizable gauge and this requires the addition of a gauge fixing lagrangian and the corresponding Fadeev-Popov ghost terms. Thus the lagrangian used in the actual calculations will be quite different from the original one; the results might also vary due to the singular nature of the unitary gauge.²³

The effects of gauge invariance at tree level calculations are also important. The fact that the effective operators are gauge invariant implies that certain vertices with a different number of legs are closely related. Consider for example, for the Standard Model gauge symmetry, the operator $\phi^\dagger \sigma_I \phi B^{\mu\nu} W_{\mu\nu}^I$ (the notation is given in Eqs. (1.2)-(1.4) and the comments following them.). This operator affects the oblique S parameter (section 8), but it also modifies the anomalous magnetic moment of the fermions, W pair production, and Higgs production at colliders; and all these contributions appear with a single coupling. There are of course many more operators contributing to these processes, but the requirement of gauge invariance does significantly restrict their number.

2.3. *Comments.*

Throughout this paper I will assume that the local gauge symmetry is $SU(2)_L \times U(1)_Y$ as in the Standard Model. The global symmetries will be taken to be the ones respected by the Standard Model.

These choices are of course not mandatory. As mentioned above, effective operators that violate the global symmetries are easily constructed (though experimental constraint require the associated scale(s) to be very large). It is also possible to choose a different gauge group. The reasonable approach is then to select a group \mathcal{G} which contains the Standard Model gauge group. This is investigated in Ref. 24 for the case $\mathcal{G} = SU(2)_L \times U(1) \times U(1)'$. The modification of the gauge group implies in general that the low energy sector is richer than in the Standard Model; for the example considered there is an additional scalar singlet (under $SU(2)_L$) and a neutral

gauge boson Z' , aside from the right handed neutrino fields required to cancel anomalies. The presence of an enlarged symmetry imposes further restrictions on the allowed operators; on the other hand the increased particle content allows for more operators to be constructed. For the model studied in Ref. 24, the second of these opposing tendencies dominates: the enlarged low energy spectrum generates a significantly larger number of operators. The phenomenological consequences of this are presented in section 8.7.

3. Effective Lagrangians.

Following the procedure outlined in the introduction one can generate an effective lagrangian by constructing, using the relevant fields, all local operators satisfying the required symmetries. The effective lagrangian \mathcal{L}_{eff} is then equal to the infinite sum of these operators multiplied by unknown (dimensionful or dimensionless) coefficients. Such an object is understood to be the low energy limit of a theory whose presence will become apparent at energies of the order of a scale Λ . All dimensionful parameters will be proportional to Λ to the appropriate power unless they are fine tuned or are protected by a symmetry; an example of the second possibility is the use of chiral symmetry to insure naturally light fermion masses.²⁵ For an excellent discussion of these issues see Ref. 1.

It is of course possible for different kinds of new physics to be generated at different scales. For example, the CP violating operators could be generated by physics whose structure becomes apparent at a scale Λ' . If this is the case then Λ will denote the smallest of such scales. It must be kept in mind, however, that some operators can have suppression factors of the type $(\Lambda/\Lambda')^n$ for some integer n .

From \mathcal{L}_{eff} we can extract those terms containing operators of (canonical) dimension ≤ 4 . The corresponding object will be denoted $\mathcal{L}^{(4)}$; $\mathcal{L}_{\text{eff}} - \mathcal{L}^{(4)}$ is an infinite series of local operators each

suppressed by the appropriate power of Λ . Some important characteristics of \mathcal{L}_{eff} will depend on whether $\mathcal{L}^{(4)}$ is renormalizable or not.

If $\mathcal{L}^{(4)}$ is renormalizable then the conditions for the decoupling theorem^{26,27} are satisfied. This insures that the contributions of the terms in $\mathcal{L}_{\text{eff}} - \mathcal{L}^{(4)}$ to any observable appear as a power series in $1/\Lambda$. In particular all effects from the dimension > 4 operators vanish as $\Lambda \rightarrow \infty$.^c The situation where $\mathcal{L}^{(4)}$ is renormalizable will be labelled the *decoupling scenario*.

The decoupling scenario is typically realized when \mathcal{L}_{eff} is obtained from the heavy physics lagrangian by letting a dimensionful parameter become very large. For example, in the limit in which the Higgs vacuum expectation value v in the Standard Model becomes very large (or equivalently when we are concerned with processes at energies significantly below the W and Z vector boson masses) we obtain QED together with an infinite tower of operators suppressed by inverse powers of v , as is the case for the four-fermi interactions.²⁸

If $\mathcal{L}^{(4)}$ is not renormalizable the decoupling theorem does not hold. The divergences generated by $\mathcal{L}^{(4)}$ cannot be absorbed in its own coefficients, and require terms from $\mathcal{L}_{\text{eff}} - \mathcal{L}^{(4)}$ in order to renormalize the theory. In many phenomenologically interesting cases this situation is associated with a derivative expansion: terms in \mathcal{L}_{eff} are collected according to the number of derivatives they carry.^d In this case we will see that there are effects which do not vanish as $\Lambda \rightarrow \infty$ despite being associated with the heavy physics (a simple example is the contribution to the oblique parameter S from a heavy generation²⁹). This situation will be denoted the *non-decoupling scenario*.

In contrast to the decoupling scenario, the non-decoupling scenario is typically realized when \mathcal{L}_{eff} is obtained by letting a dimensionless parameter become large. The reason for this difference

^c There are some contributions to Green's functions that grow with Λ , but they can all be absorbed in a renormalization of the parameters in $\mathcal{L}^{(4)}$; these contributions are important when considering the naturality of the model.

^d When fermions are included a slight modification is required, see below.

is that the constraint imposed by letting a dimensionful parameter become infinite consists in setting the corresponding field to zero. In contrast, taking a dimensionless parameter to infinity, will generate non-linear interactions among the fields.

A familiar example of the non-decoupling scenario is obtained when the scalar self-coupling constant in the Standard Model is assumed to be large. In this case we are led to a scalar sector without a Higgs particle: a non-linear (gauged) sigma model³⁰ (see section 3.2 below).

Another example for the non-decoupling scenario is generated when one or more Yukawa couplings in the Standard Model are taken to be very large. When both the Yukawa couplings of a quark doublet are taken to infinity, the low energy effective lagrangian will include a tower of non-linear scalar interactions. Among these the appropriate Wess-Zumino lagrangian³¹ is generated which insures that the resulting (effective) lagrangian is anomaly free.³² A more phenomenologically relevant investigation corresponds to the case where only one member of a quark doublet, the top, is assumed to have a large Yukawa coupling.³³ In this case the constraint on the top-bottom quark doublet is, at tree level,

$$t_L = - \left(\frac{\phi_+}{\phi_0} \right) b_L, \quad t_R = 0 \quad (3.1)$$

where $\phi_{+,0}$ are the components of the scalar doublet: $\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$. The full set of operators present in \mathcal{L}_{eff} including tree and one loop top effects can be found in Ref. 33. In this case, since the underlying theory is described by the Standard Model lagrangian, the coefficients of all the effective operators can be determined explicitly.

Further insight into these two cases can be gleaned by using a simple power counting argument. Consider a theory whose vertices are labelled by an index n ; the vertex of type n will have b_n bosonic lines, f_n fermionic lines and d_n derivatives. The dimension of an L loop integral (excluding dimensionful couplings) with I_B internal boson lines, I_F internal fermionic lines and V_n vertices of type n , is $D = 4L - 2I_B - I_F + \sum V_n d_n$. In order to renormalize the divergences

appearing in this graph we need local counterterms of dimension $\leq D$ (whenever $D \geq 0$). Using the relations

$$\sum b_n V_n = 2I_B + E_B, \quad \sum f_n V_n = 2I_F + E_F, \quad L = 1 + I_B + I_F - \sum V_n, \quad (3.2)$$

(E_B and E_F denote the number of external bosonic or fermionic lines respectively), the expression for D implies

$$S(u) = (4 - u)L + \sum_n V_n s_n(u), \quad (3.3)$$

where

$$\begin{aligned} s_n(u) &= d_n + \left(\frac{1}{2}u - 1\right) b_n + \frac{1}{2}(u - 1)f_n - u; \\ S(u) &= D + \left(\frac{1}{2}u - 1\right) E_B + \frac{1}{2}(u - 1)E_F - u; \end{aligned} \quad (3.4)$$

and u is an arbitrary (real) parameter. I will call $s_n(u)$ the “*index*” of the vertex of type n .

This treatment of power counting, though somewhat unconventional due to the presence of u , has the advantage of unifying several interesting situations to be discussed next. For example recall that power counting in the chiral approach to the strong interactions¹⁰ differs from the power counting used in evaluating ultra-violet divergences. Both these cases can be dealt with by appropriate choices of u .

Equation (3.4) implies that the L loop graphs under consideration generate vertices of index $S(u)$. This, in its turn, implies that the natural size of the coefficients of the vertices with index $S(u)$ in \mathcal{L}_{eff} are of the same order as those obtained from these graphs.

If the parameter u is chosen so that $S(u) \geq s_n(u)$ in (3.4) (for any graph and for any L), and the terms in \mathcal{L}_{eff} are ordered according to their index, that is

$$\mathcal{L}_{\text{eff}} = \sum_{i=0}^{\infty} \mathcal{L}_i \quad \text{index}(\mathcal{L}_{i+1}) > \text{index}(\mathcal{L}_i); \quad (3.5)$$

then (3.4) also insures that this hierarchy is consistent with the loop expansion: the index of a graph is never smaller than the indices of its vertices.

For example, in the non-decoupling scenario below, we will find it useful to choose $u = 2$. Then the index is independent of the number of bosonic legs: $s_n(2) = d_n + f_n/2 - 2$ and $S(2) = 2L + \sum V_n s_n(2)$, which will be used below. This relation implies the following. Suppose \mathcal{L}_0 in (3.5) contains terms with $s_n(2) = 0^e$, then the one loop graphs generate terms with index $s_n = 2$. The two loop graphs with vertices in \mathcal{L}_0 together with the one loop graphs containing one index $s_n = 2$ vertex (generated by \mathcal{L}_1) generate vertices with index $s_n = 4$, etc. For the purely bosonic terms in \mathcal{L}_{eff} the ordering (3.5) is equivalent to the derivative expansion familiar from the chiral lagrangian approach to low energy strong interactions.^{9,35,10,36,37}

Choosing now $u = 4$ gives $S(4) = \sum V_n s_n(4)$ where $s_n(4) + 4$ is just the canonical dimension of the vertex of type n ; this choice will be relevant for the decoupling scenario. In this case the terms in \mathcal{L}_{eff} are ordered according to their dimension or, for the terms in $\mathcal{L}_{\text{eff}} - \mathcal{L}^{(4)}$, as a power series in $1/\Lambda$. Then the index of a graph is equal or smaller than the power of $1/\Lambda$ of the effective operators present in the diagram. For example, graphs with a single insertion of a dimension six operator will have indices ≤ 2 which implies that the corresponding divergences (if any) will renormalize operators of index $s_n \leq 2$, *i.e.*, of dimension ≤ 6 .

Other choices for u , such as $u = 1$ (for which the ordering is independent of the number of fermionic legs) will not be examined further. I will concentrate in the two cases $u = 4$ and $u = 2$.

This discussion is necessary when effective lagrangians are to be used in perturbative calculations since an ordering consistent with the loop expansion is then required. It is possible to extend these considerations to other non-perturbative expansions, such as the large N approach (for a review see Ref. 34). For simplicity these possibilities will not be considered here.

^e The terms with $d_n = 0$ and $f_n = 2$, corresponding to the fermion mass terms in \mathcal{L}_{eff} , require special considerations, see below.

3.1. *The decoupling scenario.*

In this subsection I will study the decoupling scenario when $\mathcal{L}^{(4)}$ equals the Standard Model lagrangian, and the symmetries in \mathcal{L}_{eff} are those of the Standard Model (both local and global). The set of operators of dimension ≤ 6 constructed using the Standard Model fields was given long ago in Refs. 38, 39, 40. Given the (assumed) absence of right-handed neutrinos, it is easy to show that there are no operators of dimension five due to the various (global) symmetries. There are 81 independent operators of dimension six,^f in the sense that all other operators of dimension six are either a linear combination of them, or differ by terms which do not affect the S-matrix (see section 5 for a discussion of this last point). The whole collection can be found in Refs. 40; in appendix A the operators relevant for the three gauge boson couplings are discussed. As a simple example I present here those operators which modify the S and T oblique parameters⁴¹ (the notation is given in Eqs. (1.2)-(1.4) and the comments following them.)

$$\begin{aligned} S : \quad \mathcal{O}_{WB} &= \phi^\dagger \sigma_I \phi W_{\mu\nu}^I B^{\mu\nu}; \\ T : \quad \mathcal{O}_\phi^{(3)} &= |\phi^\dagger D_\mu \phi|^2. \end{aligned} \tag{3.6}$$

3.2. *The non-decoupling scenario.*

When $\mathcal{L}^{(4)}$ is non-renormalizable the situation which I will consider, as mentioned previously, corresponds to the case where the terms in \mathcal{L}_{eff} are ordered according to their value of $d_n + f_n/2 - 2$ (where d_n is the number of derivatives and f_n the number of fermion fields), corresponding to the choice $u = 2$ in (3.4). For the purely bosonic sector this corresponds to a derivative expansion and suggests that this is a relevant ordering when the scalar sector comprises only the Goldstone bosons that generate the masses for the gauge bosons (recall that the Goldstone bosons couple derivatively). This is further supported by the fact that in this case the scalar sector contains vertices with an arbitrarily large number of Goldstone fields (see below).

^f For one family of fermions.

Of the possible applications of this ordering, there are, as already mentioned, two cases of phenomenological interest. In the first the low energy particle content is the same as that for the Standard Model with the exception of the Higgs particle, which is assumed to be absent or heavy.³⁰ In the second case the top quark is assumed to be heavy and the Standard Model particle content, including the Higgs, generates the low energy spectrum.³³ Of course these possibilities can be mixed into the heavy top + heavy Higgs scenario. The heavy top case was briefly described above, I refer the reader to Refs. 33, for a full discussion. I will concentrate on the heavy Higgs scenario for the rest of this section.

To motivate the form of the heavy Higgs Lagrangian recall that one can describe the scalar sector in the Standard Model using the matrix

$$\Omega = (\phi, \tilde{\phi}), \quad (3.7)$$

where $\tilde{\phi} = i\sigma_2\phi^*$. It is always possible to write

$$\Omega = \frac{H+v}{\sqrt{2}} U, \quad (3.8)$$

where H is the Higgs field, v its vacuum expectation value, and U is a unitary matrix constructed from the Goldstone boson fields π^i ,

$$U = \exp\left(i\frac{\boldsymbol{\sigma} \cdot \boldsymbol{\pi}}{2f}\right), \quad (3.9)$$

where f is a constant (the “decay” constant) with units of mass. Using the wave function renormalization freedom of the π^i I can take $f = v$. (see for example Ref. 28). U transforms under $SU(2)_L \times U(1)_Y$ as

$$U \rightarrow V_L U e^{i\sigma_3\alpha/2} \quad (3.10)$$

where $V_L \in SU(2)_L$ and α is the parameter of the $U(1)_Y$ transformation.

If the Higgs particle is removed then, up to a multiplicative constant, $\Omega \rightarrow U$. One can now forget the above derivation and *postulate* that the low energy particle spectrum comprises the Standard Model fields with U replacing the scalar doublet. This is the most economical way of describing the symmetry breaking sector without any remaining physical scalar excitations.⁹ The penalty for this simplicity is the non-renormalizability of the resulting theory which implies that the low energy effective lagrangian must contain all operators involving U , the Standard Model fermion and gauge fields, and their derivatives.³⁰ The fact that the vertices of \mathcal{L}_{eff} involve U , which contains arbitrary powers of the Goldstone fields, implies that the ordering in (3.5) should be such that the index is independent of the number of boson legs, hence the requirement $u = 2$ in (3.4).

The operators of index zero without fermion fields are

$$\text{tr} \left\{ (D_\mu U)^\dagger (D^\mu U) \right\}, \quad \text{tr} \left\{ \sigma_3 (U^\dagger D_\mu U)^2 \right\}, \quad (3.11)$$

whose significance is most obvious in the unitary gauge: if we set $U = 1$ then the first operator gives the mass terms for the gauge bosons. The second operator gives a term violating the custodial symmetry⁴³ and generates a modification of the oblique T parameter, see appendix B.

The operators of index zero and two fermion fields are the fermion kinetic energy terms

$$\bar{\psi} \not{D} \psi. \quad (3.12)$$

There is also a series of operators containing two fermions together with $D_\mu U$,⁴⁴ these are given in appendix B. Note that there are terms which apparently have index $s_n = -1$: the fermion mass terms in \mathcal{L}_{eff} ,

$$m \bar{\psi}_L U \psi_R, \quad (3.13)$$

which would ruin the consistency between the ordering in \mathcal{L}_{eff} and the loop expansion. These

⁹ This choice is made for simplicity only; there is no loss of generality since all other possibilities are equivalent.⁴²

terms appear always together with the covariant derivative terms (3.12) and so it is consistent to assume that the mass parameter m has index one, which I will henceforth adopt. This is expected to be qualitatively correct as long as m can be kept naturally light (imposing, for example, a chiral symmetry). Then (3.13) will have index zero. This assumption is further discussed in Ref. 35.

The operators of index zero with four fermion fields and no U fields can be found in Ref. 40. The remaining possibility corresponds to the four-fermion operators containing one or more U fields. These can be constructed from the previous four-fermion operators by judicious insertions of the object $U\sigma_3 U^\dagger$.

There are 19 operators of index two with no fermions⁴⁵ given in appendix B. These have canonical dimension = 4 and their coefficients are Λ independent (up to logarithmic corrections). At tree level their effects do not vanish as $\Lambda \rightarrow \infty$ despite their being generated at this scale.

3.3. *Comments.*

It is worth emphasizing that the expansion of \mathcal{L}_{eff} according to the index of the operators is very different depending on the choice of u . For example, in the decoupling scenario the operator $W_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K \epsilon_{IJK}$ (the notation is given in Eqs. (1.2)-(1.4) and the comments following them.) has index $s_n = 2$ and is included in the catalogue of Ref. 40. In contrast, for the non-decoupling scenario, this operator, having six derivatives (recall that each field tensor is interpreted as a derivative commutator and so counts as two derivatives) and no fermion fields, has index four and is not included in the list of appendix B, being a higher order correction than those operators generated by $\mathcal{L}^{(4)}$ at one loop.

A natural question to ask regarding the range of applicability of the effective lagrangians described above is how close to Λ can we trust the parametrization in terms of effective local operators.

For the decoupling scenario this depends on the characteristics of the underlying physics, namely, whether the heavy excitations have wide or narrow resonances. If wide, these resonances will have long “tails” and their effects will intrude into what one would naively consider the region where the effective lagrangian parametrization should be valid. This generates anomalously large coefficients (up to an order of magnitude in some cases) for some of the operators, compared to the values expected from naturalness (see section 4.2).

In the non-decoupling scenario the above arguments indicate that, for those terms without fermions, \mathcal{L}_{eff} is organized as a (covariant) derivative expansion. We will see in the next section that the coefficients of an n derivative operator is proportional to $1/(4\pi v)^n$. In the case of the Higgs-less Standard Model $v \simeq 246$ GeV so that the effective lagrangian parametrization will certainly break down at energies of the order $4\pi v \sim 3$ TeV in this case.

The question now arises as to the significance of this scale. It was argued in Ref. 46 that, since \mathcal{L}_{eff} will generate all required physical amplitudes with the correct unitarity cuts as long as the energies are below $4\pi v$, the breakdown of this expansion must be associated with a new mass threshold at or below this scale. In this argument strong use is made of the naturalness requirement³ (see section 4.1) that \mathcal{L}_{eff} is an expansion in $\partial/(4\pi v)$. Reference 46 also provides various examples in which these arguments are illustrated. The arguments can also be applied to low energy pion physics, where the scale $4\pi v$ is remarkably close to the mass of the ρ meson.⁴⁷

These statements have been criticized in Ref. 48 on the basis that the breakdown of the expansion in a derivative series only implies that one must sum terms of all orders before deriving any conclusions, and that there are functions (which may represent Green’s functions) for which the derivative expansion has large coefficients but still may be summed into an analytic result. The derivative expansion in the example provided, however, does not have the same order of magnitude coefficients as required by the arguments in Refs. 46, 47, but are instead much smaller.

4. Orders of Magnitude.

In this section I will discuss various issues surrounding the magnitude of the effects produced by operators of dimension greater than four. I will first consider the expected order of magnitude for the coefficients of the effective operators in the non-decoupling scenario. Then I will turn to the same question in the decoupling scenario.

4.1. *The non-decoupling scenario.*

In order to determine the order of magnitude of the coefficients of the operators presented, for example, in appendix B, it is natural to require that the radiative corrections to the coefficient of an operator be at most as large as its tree level value. Equivalently one can argue as follows:^{3,49} when considering the renormalization of the operators in \mathcal{L}_{eff} it is found that all the coefficients are in general renormalized. It is then understood that the renormalization group running of these quantities can be used to evolve them in energy, up to the scale where a mass threshold is crossed (presumably at a scale $\sim \Lambda$).⁴ At this point the effective coefficients are determined by the couplings of the heavy theory. Then it is natural to require that, if the renormalization scale is changed by a factor of order one, the running parameters also change by a factor of order one. This requirement (in either of its forms) determines the order of magnitude for the coefficients; for a discussion on the assumption of naturality see section 7.3.

The procedure which I will follow to determine the coefficients of \mathcal{L}_{eff} appeared in Ref. 50. I will assume that the theory is explicitly gauge invariant, either because it was originally so, or because it was rendered gauge invariant using the procedure outlined in section 2.1.

Consider a theory with scalar fields ϕ , fermionic fields ψ and gauge bosons W . Then the relevant vertices have the symbolic form

$$\Lambda^4 \lambda (\phi/\Lambda_\phi)^A (\psi/\Lambda_\psi^{3/2})^B (p/\Lambda)^C (gW/\Lambda)^D, \quad (4.1)$$

where p represents a derivative, Λ is a UV cutoff, λ is a coupling constant, and the other scales, $\Lambda_{\phi, \psi}$, are to be determined. Since Λ is associated with the momentum scale I divide p (a generic momentum) by this scale; since gauge fields appear only in covariant derivatives, they are divided by the same scale. The quantities A , B , C and D are assumed to be integers. Since the W fields appear always inside a covariant derivative it is sufficient to consider vertices with $D = 0$ (field tensors are treated as covariant derivative commutators).

I will assume that the gauge-boson couplings to fermions and bosons (including self couplings) are $\lesssim 1$; based on this assumption I neglect internal gauge boson lines, as well as Fadeev-Popov ghost lines (this refers to the Standard Model gauge bosons, the ones of the underlying theory — if any — are already “integrated out”). Note that since we are dealing with a gauge theory, I can choose a gauge in which the W propagator which drops off at large momentum.

Now consider a graph with V vertices which generates an L loop correction to (4.1). This contribution will be of the same order provided (I replace all loop momenta by Λ since we are interested only in an order of magnitude estimate)

$$1 \sim (\Lambda^4 \lambda)^{V-1} \Lambda_{\phi}^{A-\sum A_i} \Lambda_{\psi}^{3(B-\sum B_i)/2} \Lambda^{C-\sum C_i} \Lambda^{\sum C_i - C + 4L - 2I_{\phi} - I_{\psi}} (4\pi)^{-2L}, \quad (4.2)$$

where i labels the vertices in the graph and I_{ψ} (I_{ϕ}) is the number of internal fermion (boson) propagators. Using the relations $\sum A_i = A + 2I_{\phi}$ and $\sum B_i = B + 2I_{\psi}$ (4.2) becomes

$$(16\pi^2 \lambda)^{-L} \left(\frac{\lambda \Lambda^2}{\Lambda_{\phi}^2} \right)^{I_{\phi}} \left(\frac{\lambda \Lambda^3}{\Lambda_{\psi}^3} \right)^{I_{\psi}} \sim 1; \quad (4.3)$$

this requires

$$\lambda \sim \frac{1}{16\pi^2}; \quad \Lambda_{\phi} \sim \frac{1}{4\pi} \Lambda; \quad \Lambda_{\psi} \sim \frac{1}{(4\pi)^{2/3}} \Lambda. \quad (4.4)$$

Substituting back into (4.1), and using the fact that gauge bosons appear always in a covariant

derivative denoted by \mathcal{D} , yields

$$\frac{\Lambda^4}{(4\pi)^{2-A-B}} \left(\frac{\phi}{\Lambda}\right)^A \left(\frac{\psi}{\Lambda^{3/2}}\right)^B \left(\frac{\mathcal{D}}{\Lambda}\right)^C. \quad (4.5)$$

If the scalars appear in the form of a unitary field,

$$U \sim \exp(i\phi/\Lambda_\phi) \sim \exp(4\pi i\phi/\Lambda), \quad (4.6)$$

then the vertex takes the form

$$\frac{1}{(4\pi)^{2-B}} \Lambda^4 \frac{\mathcal{D}^C U^{A'} \psi^B}{\Lambda^{C+3B/2}}, \quad (4.7)$$

where A' is an integer and, as above, \mathcal{D} denotes the covariant derivative. Note that, as claimed in section 3.2, this implies that the bosonic sector of \mathcal{L}_{eff} is organized as an expansion in powers of \mathcal{D}/Λ .

The operators in (3.11) have coefficients of order $(\Lambda/4\pi)^2$. The first of these operators, $\text{tr} |D_\mu U|^2$ corresponds to $B = 0$, $A' = C = 2$, and generates the masses of the gauge bosons (this is obvious in the unitary gauge). It then follows that

$$\Lambda \simeq 4\pi v \sim 3 \text{ TeV} \quad (\text{non-decoupling scenario}); \quad (4.8)$$

where $v \simeq 246 \text{ GeV}$ corresponds to the scalar vacuum expectation value in the Standard Model. The other operator in (3.11) should also have a coefficient of order v^2 according to the previous argument; in fact, experimentally it is found that its coefficient is very much suppressed, being proportional to the deviation of the ρ parameter from one. This illustrates a caveat in the above estimates: the results (4.7) corresponds to the largest radiative corrections allowed by naturality arguments. But in specific situations the coefficients can be suppressed due to some unknown effects in the underlying theory. Such a situation is envisaged in Ref. 51 where the heavy physics is assumed to respect the $SU(2)_R$ symmetry of the scalar sector in the Standard Model, with this assumption all operators of dimension ≥ 6 violating this symmetry can be ignored.

The operators in (3.12) have coefficients of order one. Then, by the discussion below this equation, the same will be the case for the operators in (3.13). Note that it is possible to impose a chiral symmetry to protect the fermion masses; in this case one can naturally assume that they are $\ll \Lambda$.²⁵

Similarly the four-fermion operators have coefficients of order $(4\pi/\Lambda)^2 \simeq 1/v^2$. This is different from the usual assumptions^{52,53,54,55,56,57} where the coefficient is taken to be $4\pi/\Lambda^2$. This is argued on the basis that the coefficient is of the form g_H^2/Λ^2 for some coupling g_H ; assuming the underlying theory is strongly interacting implies g_H^2 is large, and this is presumed to mean that $g_H^2 = 4\pi$, *i.e.* $\alpha_H = 1$, which taken as the *definition* of a strongly interacting theory. This is of course a fuzzy statement: one could equally define a strongly interacting theory by the condition $\alpha_H = 4\pi$, which is further supported by the arguments above. Note however that the assumption $\alpha_H = 1$ does not necessarily contradict the above naturality value for the coefficients since, as mentioned above, they can be suppressed.^h Finally note that all index $s_n = 2$ operators in appendix B have coefficients of order $1/16\pi^2$.

These estimates can be carried over to the one example of a strongly coupled theory to which this approach has been applied, and for which there are abundant experimental results: the low energy strong interactions. The results (see Ref. 10 for a recent review) are perfectly consistent with the above estimates (for example, the *measured* values for the coefficients of the four derivative operators are all of order $1/16\pi^2$).

^h A more serious problem arises when the kinetic energy for the gauge bosons is considered. This corresponds to $A' = B = 0$ and $C = 4$ in (4.7) which implies a coefficient $\sim 1/16\pi^2$ instead of the correct $1/4$; see Ref. 50 for a discussion on this point.

4.2. *The decoupling scenario: tree vs. loop-generated operators.*

Strongly coupled case

In the decoupling scenario and when the underlying physics is strongly interacting, one might attempt to follow the same arguments as in the previous section and assert that (4.5) should remain valid. There is a serious problem with this reasoning: if we consider the mass terms for the scalars, corresponding to $A = 2$, $B = C = 0$ in (4.1), it follows immediately that the masses are $O(\Lambda)^i$. The scalar fields should, in this case, not be included in the low energy spectrum,^j and we are lead back to the non-decoupling scenario.

It is possible to devise models where there are cancellations among diagrams thus allowing $m_{\text{scalar}} \ll \Lambda$; an example is given by imposing supersymmetry on the model.²¹ Such theories invariably generate light particles not present in the Standard Model and will be probed directly in the next generation of colliders; their study using an effective lagrangian approach requires then the modification of the low energy spectrum. In order to keep the discussion at a manageable level I will ignore this possibility in the following. Thus I will assume in the remainder of this paper that the decoupling scenario is associated with a weakly interacting underlying theory, which I discuss next.

Weakly coupled case

For the decoupling scenario, and when the underlying theory is weakly coupled the considerations used previously to determine the operator coefficients are not applicable due to the assumption that the couplings are small: higher loops only give small corrections. The relevant question now becomes, which operators can be generated at tree level by the underlying theory?

ⁱ The same appears to be true for the fermion masses. This disaster can be avoided imposing a chiral symmetry which insures that, for the case $B = 2$, $A = C = 0$, (4.1) will acquire a factor m/Λ , where m is the tree level fermion mass.

^j This problem does not arise for the non-decoupling scenario due to the Goldstone nature of the scalar fields in that scenario.

This is so because loop-generated operators will acquire a loop suppression factor of $\sim 1/16\pi^2$ which is absent for the tree-level operators (this will be discussed further below), hence their effects are quantitatively much smaller. It must also be kept in mind that the operator coefficients will also contain small ($\lesssim 1$) coupling constants.

In the discussion below I will consider the interesting case where the low energy physics is described by the Standard Model and assume that the heavy physics is described by a gauge theory. Before proceeding I will standardize notation. In the full theory I separate the gauge indices corresponding to the low energy gauge (Standard Model) group, denoted by a, b , etc., from the remaining gauge indices, denoted by A, B , etc. The structure constants of the group in the full theory are denoted by f . Light gauge bosons are denoted by W , heavy ones by \mathcal{W} ; light scalars are denoted by ϕ , heavy scalars by Φ ; similarly light fermions are denoted by ψ while their heavy counterparts by Ψ .

Formally what is being done is “to integrate out”, to lowest order in \hbar , all the heavy fields. The gauge fixing in the heavy theory is such that the resulting effective action is manifestly gauge invariant.⁵⁸ Thus, for this calculation, all light fields are external and hence I can assume that the light indices are unbroken (the light gauge group is the unbroken subgroup). The broken generators (those with indices A, B , etc.) carry a representation of the unbroken group (see for example Ref. 59). In the calculations below I will assume for simplicity that the heavy physics has no super-renormalizable vertices, and I will use the fact that there are no dimension five operators that can be constructed from Standard Model fields satisfying the Standard Model symmetries; for a full discussion of the general case see Ref. 60.

The gauge structure of the full theory disallows certain vertices. Consider first a vertex with two light gauge bosons and one heavy one, it is proportional to f_{abC} which vanishes since the unbroken generators form a Lie algebra: the $W^2\mathcal{W}$ vertices are absent. Similarly there are no $W^3\mathcal{W}$ vertices. Vertices of the type $W^2\Phi$ or $W^2\phi$ are also not present since they contain two

unbroken generators and they must be proportional to a vacuum expectation value. Vertices of the type $W^2\phi\Phi$ and $W\psi\Psi$ are also not allowed. Finally vertices of the type $WW\Phi$ are absent since the fields Φ are orthogonal to the Goldstone boson directions.

With these restrictions it is easy to see that there are no tree level graphs with only W external legs corresponding to operators of dimension six. For graphs with some ψ or ϕ external legs I use the fact that the final result will be gauge invariant in order to consider only those graphs with no external W legs: these will be generated by replacing derivatives with covariant derivatives. It is then a matter of patience to prove that the only tree-level operators of dimension six (for a theory with super-renormalizable vertices the same results are obtained⁶⁰) are

$$\begin{aligned} &(\phi D\phi)^2, \quad (\phi\bar{\psi})\not{D}(\phi\psi), \quad \phi^3\bar{\psi}\psi \\ &(\bar{\psi}\psi)^2, \quad \phi^6; \end{aligned} \tag{4.9}$$

(understood to be a schematic description of the relevant operators).

The operators of the type (4.9) will be suppressed only by two coupling constants: in $(\bar{\psi}\psi)^2$ the coupling for the vertex $\bar{\psi}\psi\Phi$ occurs twice; in the remaining operators containing fermions the coupling for the vertex $\bar{\psi}\phi\Psi$ occurs twice; in ϕ^6 the coupling of the vertex $\phi^3\Phi$ appear quadratically, etc. Among the operators in Ref. 40 all but the above will have coefficients $\sim 1/16\pi^2$ (see section 4.4).

Higher dimensional operators

Heretofore I have concentrated the discussion on dimension six operators. There are, however, situations where higher dimensional operators are non-negligible and can even be dominant.⁶⁰ This occurs if, for the decoupling scenario, there are contributions to a certain observable from both loop-generated dimension-six operators, and tree-generated dimension-eight operators. The suppression in the first case is $\sim 1/16\pi^2$, while in the second it is $\sim v^2/\Lambda^2$, where v is the Standard Model scalar vacuum expectation value. It follows that for scales below $4\pi v \sim 3$ TeV

one must include all contributions from tree level generated dimension eight operators in any calculation for which the contributing dimension six operators are loop generated.

4.3. *Predictability in effective theories.*

The often-claimed defect of theories with terms of mass dimension greater than four lies, not in their lack of renormalizability (since they *are* renormalizable, see section 7), but in the presumed lack of predictability. This is in fact not the case.

To see this recall that, as was emphasized in section 3, it is possible to order the terms in \mathcal{L}_{eff} according to their index, this ordering being consistent with the loop expansion. Suppose now a certain set of observables is calculated using only operators whose indices are below a certain upper value. It is then clear that we are dealing with a theory with a finite number of parameters and so with non-trivial content. Moreover one can use (4.1) or (4.7) to estimate the corrections generated by terms of higher indices;¹ the corrections are, within the range of applicability of the effective lagrangian parameterization, subdominant. The number of terms in the expansion that must be kept depends on the level of precision the experiment has achieved. Similar arguments can be used in the decoupling scenario.

Thus, despite having an infinite set of parameters, effective theories do have content (a fact routinely used in the chiral lagrangian approach to the strong interactions at low energies^{9,10}). Effective lagrangians are provisional models whose parameters we expect to be able to deduce from a small set of constants appearing in a more fundamental theory; but, as long as we have no direct evidence of the new interactions, this way of parametrizing is very appealing as it is theoretically consistent and model and process independent.

4.4. *Naturality in effective theories.*

It could be argued that naturality is in itself an extra assumption which has no scientific origin, but an æsthetic one. If one disposes of naturality on the basis of being as open minded as possible, then one can include any (Lorentz invariant) term in \mathcal{L}_{eff} with completely arbitrary couplings.

This type of models is useless since it completely lacks predictive power. Since the model has absolutely no constraints on the coefficients, no relationships are obtained between the various terms in the lagrangian; the fact that experiments confirm various connections between such coefficients must be assumed to be coincidences within this approach. For example the fact that $\rho = 1$ at tree level in the Standard Model would be an interesting fact, but of no deep significance since the effective lagrangian would have terms of the form $m_W^2 W^+ \cdot W^-$ and $m_Z^2 Z^2/2$ with no connection between m_W , m_Z and the weak mixing angle derived, for example, from the neutrino cross sections; even if this connection is imposed at tree level, it will be spoilt, and very strongly, by loop corrections. Similar arguments could be constructed, for example, for lepton universality.

The point is that one cannot have it both ways: if the Standard Model is assumed as an excellent first approximation, then the corrections should be such that this assumption remains valid. This requires the presence of gauge invariance and a hierarchy on the coefficients of the effective operators. If either of these conditions is violated the consistent evaluation of all low energy processes will generate a theory different from the one originally considered. For example, most or all of the masses would receive corrections of $O(\Lambda)$ (sections 4.5 and 7.3), and the couplings become large (section 7.3).

4.5. *Loop factors.*

In this sub-section I re-examine the claims made above that the coefficients of loop-generated operators are subdominant. This discussion will be limited to the case where the heavy physics is weakly coupled.

It is clear that, generically, any single loop contribution is accompanied by a factor of order $1/16\pi^2$. One must also remember that the heavy loops will contain not only these loop factors but also a certain number of small coupling constants. Still, it is often argued that this suppression can be reduced when many graphs contribute to the same physical quantity, provided the contributions add coherently. One could envisage, for example, integrating out N identical heavy fermions with $N \gtrsim 160$, in which case the total loop contributions would be unsuppressed. The simplest view one can take for this type of situation is that, since tree-level and one-loop contributions are comparable, the loop expansion is not useful in analyzing this type of model, which are not manageable using perturbative techniques (note that it is not assumed that the couplings are suppressed by factors of $1/N$ and so the large N expansion will also not be useful).

Of particular relevance are the contributions from this large number of particles to the vacuum polarization of the light excitations. From the fact that the gauge boson masses are protected à la Veltman,⁶¹ it follows that the presence of $\sim 16\pi^2$ virtual loops adding coherently will give a large correction, of order 100% to the gauge boson masses (see section 7.3 for a related discussion). The scalar masses, in contrast, are not necessarily protected and a correction of order Λ is expected (see the example below for a specific model). It follows that under these circumstances all scalars coupling to a large number of heavy fields would become heavy through radiative corrections, radically modifying the low energy theory. An exactly solvable example of this situation is presented next.

A simple example.

To see whether these somewhat qualitative arguments are quantitatively correct, it is convenient to study an exactly solvable model. I chose a theory of N degenerate fermions interacting with a scalar field θ in two space-time dimensions.

The lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial\theta)^2 + i \sum_{a=1}^N \bar{\psi}_a \mathcal{D}\psi_a; \quad \mathcal{D} = \not{\partial} + \frac{ig}{2}\gamma^\mu(\partial_\mu\theta) - im e^{ig\theta\gamma_5} \quad (4.10)$$

where g is a coupling constant and m a heavy mass.

I will be interested in the effective theory obtained by integrating out the fermions. A chiral rotation in the fermions changes θ and induces a Jacobian, which is evaluated using Fujikawa's method.¹¹⁴ The result, described in appendix C, is

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\chi)^2 + \frac{Nm^2}{4\pi} \cos(\lambda\chi); \quad \chi = \frac{2g\theta + \pi}{\lambda}, \quad \lambda = \frac{2g}{\sqrt{1 + g^2N/(4\pi)}} \quad (4.11)$$

This is the Sine-Gordon model extensively studied in the literature (for a review see Ref. 62). The mass of the linear excitations is $2m/c$ where $c = \sqrt{1 + 4\pi/(g^2N)}$, the solitons present in this model have mass mNc/π ,⁶² etc. Since $c \geq 1$ the solitons are always heavy; light scalar states appear only when $c \gg 1$, which requires $g^2N \ll 4\pi$. In the limit where N offsets the loop suppression factor $g^2/4\pi$, the quantity c is of order one and there are no light excitations at all.

These support the previous claims that whenever the number of graphs is so large as to compensate the loop suppression factor (including the small coupling constants) a strongly coupled theory results; in this case the loop expansion is not reliable. The presence of a large number of particles in the loops can indeed overcome the loop suppression factors, but in this case the low energy theory is completely different from the one naively expected.

5. Equations of Motion and Blind Directions.

5.1. EQUATIONS OF MOTION

The classical equations of motion can be used to reduce the number of operators in \mathcal{L}_{eff} .^{40,6,63} This is based on the fact that if two operators \mathcal{O} and \mathcal{O}' are such that $\mathcal{O} - \mathcal{O}' = \mathcal{A}(\delta S / \delta \chi)$, where \mathcal{A} is some local operator, S is the classical action and χ a field of the theory, then an insertion of \mathcal{O} gives the same S-matrix element as that of \mathcal{O}' (though the Green's functions *are* different). A simple proof of this fact is presented in appendix D; for a thorough discussion see Ref. 63. Such operators will be called *equivalent*.

Thus, if \mathcal{L}_{eff} has a term $\eta \mathcal{O} + \eta' \mathcal{O}'$ where \mathcal{O} and \mathcal{O}' are equivalent, then, to lowest order in η and η' , all physical quantities will depend on $\eta + \eta'$ but never on either of them independently. A term in \mathcal{L}_{eff} is *redundant* if it is equivalent to another term already in the effective lagrangian. In Ref. 40 and many other publications great efforts are made to eliminate all redundant operators.

There is an important point which is obscured by eliminating redundant operators. Consider for example the case where the heavy physics is weakly coupled, and suppose \mathcal{O} and \mathcal{O}' are equivalent. It is then possible for the heavy physics to generate, for example, \mathcal{O} at tree level, and \mathcal{O}' at one loop. Thus if we eliminate \mathcal{O} in favor of \mathcal{O}' and estimate that coefficient for \mathcal{O}' has its natural size, we would severely underestimate the heavy physics effects : there is in fact a very significant contribution from \mathcal{O} unencumbered by loop suppression factors.

As a concrete example consider the operator

$$\{(D_\mu \phi)^\dagger (D_\nu \phi) + \frac{1}{2} \phi^\dagger [D_\mu, D_\nu] \phi\} B^{\nu\mu} \quad (5.1)$$

which, in the decoupling scenario for a weakly interacting heavy theory, is generated only via loops (see section 4.2). On the other hand the use of the equations of motion show that it is equivalent to $(\phi^\dagger D^\nu \phi) j_\nu^{(B)}$ where $j_\nu^{(B)}$ is the source current for B ; this last operator can be

generated at *tree* level. Therefore, even if the S-matrix elements cannot distinguish between the first and second operators, there is a very large quantitative difference whether the underlying physics generates the second one or not.

In view of the above it appears to be more useful not to eliminate redundant operators from \mathcal{L}_{eff} ; then one can always estimate the contributions from any given operator without having to keep track of the expected order of magnitude of the contributions from other operators equivalent to it. One may also exploit the redundancy of operators to check calculations of S-matrix elements.

5.2. BLIND DIRECTIONS.

Another consideration relevant for quantitative estimates is the possible presence of “blind directions”:⁶ these are operators to which we have no experimental sensitivity since they affect quantities only at the one loop level (or beyond). An example is the operator

$$\mathcal{O}_W = \epsilon_{IJK} W_{\mu\nu}^I W_{\nu\lambda}^J W_{\lambda\mu}^K. \quad (5.2)$$

In some publications (see, for example, Ref. 6) it has been claimed that there is no fundamental difference between the blind operators and the “sighted” ones. One can then translate any limits on Λ obtained by using a sighted operator into limits on the effects of a blind operator. If correct, this argument implies that the limits derived from the measurements at LEP1 are so severe that LEP2 will be almost insensitive to any kind of new physics. I will now show that this claim is in fact an extra assumption strongly dependent on the heavy physics. Counter-examples are readily available.

Consider first the following toy model consisting of a light scalar field ϕ interacting with two

heavy fermions ψ_a ($a = 1, 2$). The lagrangian is

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{6}\sigma\phi^3 - \frac{1}{24}\lambda\phi^4}_{\text{light sector}} + \underbrace{\sum_{a=1}^2 \bar{\psi}_a (i \not{\partial} - M + (-)^a g\phi) \psi}_{\text{heavy sector}} \quad (5.3)$$

When the fermions are integrated out, they produce an effective action that is even in ϕ . If low energy ($\ll M$) experiments are sensitive to, for example, ϕ^5 but not to ϕ^6 (which is then a blind direction), there would be no experimental indication of the heavy sector. The null results could be interpreted as Λ being astronomical if the above assumptions are made, while in fact a new generation of experiments could very well uncover the presence of the heavy fermions.

This would have little more than academic interest if the same argument could not be applied to realistic models. In fact this is not the case. Suppose we add to the Standard Model two vector-like fermion doublets Ψ_a , $a = 1, 2$, which have a common mass $M \gg v$. Suppose one doublet has hypercharge y the other $-y$. If we integrate these fermions out we generate a series of one loop graphs with external B and W legs (the notation is given in Eqs. (1.2)-(1.4) and the comments following them.). The choice of masses and hypercharges guarantees that all graphs with an odd number of B legs will cancel out.^k This implies that, for example, there will be no contribution to the oblique S parameter,⁶⁴ see (3.6). If we then take the point of view advocated in Ref. 6, we would conclude that the scale of new physics is enormous, and that there is no hope in the near future of being sensitive to, for example, the physics that generated the operator (5.2).

There is no mystery in the above results: unknown symmetry properties of the heavy physics may forbid certain operators. If we have, accidentally, a preference towards measuring the effects of precisely the suppressed operators, we would then (erroneously) conclude that the scale of new physics is extremely high. This could be rephrased by stating that the scale responsible for a certain set of operators could be very high, but we cannot extend this to the whole manifold

^k If the fermion masses are split by an amount $\Delta M \ll M$, the terms with an odd number of B legs are suppressed by a power of $\Delta M/M$.

of possible operators; though this is often assumed, it remains just that, an assumption. When evaluating the results of a certain experimental search this fact must be kept in mind: though factors such as the loop suppression factors do restrict our sensitivity to new physics (see section 8), it is conceivable that current measurements are prejudiced, due to experimental constraints, against the strongest effects from new interactions which may be revealed in future experiments.

As a last comment on blind directions it is important to point out that one can always try to dispose of such operators by changing to a basis where most operators are “sighted”. To illustrate this point I will consider the case where the low energy theory is the Standard Model without fermions and consider the blind operator^{6,65} $\mathcal{O} = B^{\mu\nu} D_\mu \phi^\dagger D_\nu \phi$ (the notation is the same as above). Use of the equations of motion shows that this is equivalent to the linear combination

$$\frac{ig}{4} B^{\mu\nu} W_{\mu\nu}^I \phi^\dagger \sigma_I \phi + ig' |\phi^\dagger D_\mu \phi|^2 + \frac{ig'}{2} (\phi^\dagger \phi) |D_\mu \phi|^2 + \frac{ig'}{4} (\phi^\dagger \phi) B_{\mu\nu}^2 \quad (5.4)$$

plus a renormalization of the scalar self coupling constant (the notation is given in Eqs. (1.2)-(1.4) and the comments following them.). The first operator affect the oblique parameter S , similarly the second modifies T ; the third operator modifies the Fermi constant while the last one affects the Z mass and the weak mixing angle. The point is that all these operators affect measured quantities at tree level. Thus we obtain a better bound on Λ by choosing the second set of operators than the original one. This result should be tempered, however, by the previous comments on the magnitude of the coefficients of equivalent operators.

6. Unitarity.

Effective interactions vertices have been used to provide examples where tree level unitarity is violated, and this deserves comment. First it must be remembered that violation of tree level unitarity does not mean that the theory violates unitarity, but that the coupling constants are so large that perturbative calculations are unreliable. Thus, violation of tree level unitarity signals the onset of a strongly interacting regime.

It must also be remembered that the use of effective operators carries the responsibility of not using this parametrization above a certain scale Λ . The question to ask is therefore: given a certain reaction whose CM energy is below Λ , are there violations of tree level unitarity? In order to answer this, the results of section 4 must be used in order to estimate the magnitude of the coefficients of the effective operators.

For example, in the decoupling scenario, a typical operator \mathcal{O} of dimension six with ≤ 2 scalar fields, produces, in a $2 \rightarrow 2$ process, amplitudes that grow like s (s is the usual Mandelstam variable), which has the potential of violating tree level unitarity. But to actually determine whether this possibility is realized it must be remembered that the operator under consideration appears as a term $\alpha\mathcal{O}/\Lambda^2$ in \mathcal{L}_{eff} , so that its contribution to the scattering amplitude is of order $s\alpha/\Lambda^2$, where $\alpha \lesssim 1$ (see section 4). It follows that these terms can generate (tree level) unitarity violations only for energies $\gtrsim \Lambda$, *i.e.* beyond the range of applicability of the effective lagrangian parametrization (at such energies the higher dimensional operators dominate over the ones included and the expansion breaks down).

A specific example of this is the following: consider in the process $e^+e^- \rightarrow W^+W^-$, the contributions of the effective lagrangian

$$\mathcal{L}_{\text{St.Model}} - \frac{ie}{m_W^2} W_{\lambda\mu}^+ W^{-\mu}{}_{\nu} (\lambda_\gamma F^{\nu\lambda} + \cot\theta_W \lambda_Z Z^{\nu\lambda}), \quad (6.1)$$

obtained from Ref. 18; F denotes the electromagnetic field tensor, $Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu$ and

similarly for $W_{\mu\nu}^{\pm}$; $\lambda_{\gamma,Z}$ are constants and θ_W denotes the weak mixing angle. If this is rendered gauge invariant along the lines of section 2 then $\lambda_{\gamma,Z} \sim g^2/16\pi^2$ (see section 4.1).

The amplitude for $e_L^+ e_R^- \rightarrow W_L^+ W_L^-$ is¹⁸

$$\mathcal{A} = e^2 \sin \Theta \frac{\lambda_Z - \lambda_\gamma}{4m_W^4} s^2 + O(s), \quad (6.2)$$

where Θ is the W center of mass scattering angle. The corresponding $\ell = 0$ partial wave violates unitarity when

$$\sqrt{s} \gtrsim \left(\frac{128m_W^4}{e^2 |\lambda_Z - \lambda_\gamma|} \right)^{1/4} \simeq \frac{0.5 \text{ TeV}}{|\lambda_Z - \lambda_\gamma|^{1/4}} \quad (6.3)$$

If the λ term in the denominator is assumed to be of order one, then this corresponds to an energy ~ 500 GeV; when the λ have their natural sizes, this is increased to ~ 2.1 TeV, which is of the same order as the scale where the effective lagrangian parametrization breaks down (see (4.8)). It is therefore inconsistent to suppose that the coefficients λ are of order one.

Within the decoupling scenario the above lagrangian is generated by the operator (5.2). In this case however $\lambda_Z = \lambda_\gamma$ so that the potential tree level unitarity violations are generated by the $O(s)$ terms in \mathcal{A} (if at all) and will be much smaller.

This example illustrates the importance of keeping all small factors. If the coefficients in \mathcal{L}_{eff} are assumed to be unnaturally large the conclusion would be that there are very significant effects at scales well within reach of a near-future collider. If the natural size of the coefficients is retained the effects are generally found to be very small.

Another case where the study of unitarity in an effective theory becomes interesting is within the context of the so called “delayed unitarity” scenario.⁶⁶ The idea is that gauge theories often require cancellations among diagrams to enforce unitarity. If some of these diagrams are suppressed with respect to others in some energy range, these cancellations will not be apparent in the said range; this can give rise, in some cases, to measurable effects. So, for example, in

an explicit calculation⁶⁶ of the effects of a hypothetical fourth generation of heavy fermions in $e^+e^- \rightarrow W^+W^-$, the cross section acquires a correction factor $\sim 1+s/(4\pi v)^2$ where $v \simeq 246$ GeV. Since this is a perturbative calculation, it is reliable as long as perturbation theory is valid, which requires the mass of the fermions to be below ~ 0.5 TeV;⁶⁷ and since the heavy fermions are assumed not to be produced, it follows that the expressions are relevant for s below 1 TeV. In this range corrections of $\sim 10\%$ are possible. These results are of course in agreement with the estimates of section 4. Note, however, that it would be very misleading to say that the corrections are $O(s/m_W^2)$ (rather than $\alpha s/16\pi^2 m_W^2$) since this is ~ 150 ; the factors of g and 4π are very important for any reliable quantitative estimate.

More insight can be gained by considering the scattering of longitudinally polarized W vector bosons. At energies such that $s, m_H^2 \gg m_W^2, m_Z^2$ the amplitude is^{3,68,12} (for a review see Ref. 16)

$$\mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = -\sqrt{2} G_F m_H^2 \left[\frac{s}{s - m_H^2} + \frac{t}{t - m_H^2} \right] \xrightarrow{m_H^2 \gg s} -\sqrt{2} G_F u. \quad (6.4)$$

The corresponding $\ell = 0$ partial wave is given by

$$a_0 \simeq \frac{G_F}{16\sqrt{2}\pi} s, \quad m_H^2 \gg s \gg m_Z^2 \quad (6.5)$$

which grows with s and will appear to violate unitarity provided $s \sim 16\sqrt{2}\pi/G_F$ lies in the allowed range for s ; at energies $\gtrsim m_H$, however, a_0 will go to a constant.

To examine whether tree level unitarity is in fact violated note that, for the above computations to be reliable, perturbation theory must be valid. This requires that the Higgs self coupling be smaller than $\sim 16\pi^2$; so that $m_H \lesssim 4\pi v$, where v is the scalar vacuum expectation value. The region where a_0 grows linearly with s is bounded by $s \lesssim m_H^2$ which, using the above limit for m_H implies $a_0 \ll \pi/2$ so that unitarity violations are in fact not generated. If the Higgs mass is pushed beyond the above perturbative limit, unitarity is apparently violated, but this is just a signal that the higher order loop corrections are as important as the tree level contributions to a_0 : the scalar sector becomes strongly coupled.³⁰

7. Radiative Corrections.

In this section I will review the divergence structure of effective theories and show that these are renormalizable theories. The relationship of renormalizability to naturality, gauge invariance and unitarity will be investigated.

7.1. *Renormalizability.*

Effective theories, just like “ordinary” theories can be used in perturbative calculations; and just as in ordinary field theories divergences are encountered in loop calculations. Thus the model requires, as a first step, regularization; we will use the language of dimensional regularization since it is guaranteed to preserve the gauge invariance of such importance in practical calculations. This approach has the defect of hiding power divergences, but these can be recovered by considering the model in fewer dimensions.⁶¹

When studying the perturbative expansion of an observable in an effective theory a tower of divergences is obtained whose structure is more complicated than the one found in usual renormalizable theories. These divergences have, however, two important properties: *(i)* they correspond to local interactions; and *(ii)* these local interactions respect all the symmetries of the theory, assuming that they are preserved in the (dimensionally or otherwise) regularized theory.

The first property can be proved just as in the usual renormalizable theories.^{69,27} Consider any graph G depending on some external momenta, collectively denoted by p : $G = G(p)$; if we now take sufficient p derivatives of G , the result will be a finite integral. It follows that G can be written as a polynomial in p with divergent coefficients plus a finite part. If we now sum all graphs relevant for a given process, the result will have the same characteristics. The divergent terms, being polynomial in the external momenta, can be associated with a set of local operators satisfying the symmetries of the model. These terms can be absorbed in renormalizing

the coefficients of \mathcal{L}_{eff} since, by definition, the effective lagrangian already contains all such operators. Thus effective theories *are* renormalizable. This fact has been used repeatedly in the chiral approach to strong interactions;^{3,9,10} in the non-decoupling scenario,^{70,65,6} and in the non-decoupling scenario.^{6,71,65}

This argument has another application. The presence of effective operator insertions in loops implies that the effective lagrangian parametrization is being used at momentum transfers much larger than Λ , and this may appear inconsistent. To justify the results obtained in this manner⁷¹ note that, as mentioned above, taking a sufficient number of derivatives any graph G can be rendered finite; the resulting integral has then a cut-off equal to the largest mass in the loop²⁷ and this scale is $\ll \Lambda$. Therefore the effective operators can be inserted safely in $\partial_p^n G(p)$ for sufficiently large n . Integrating then yields the above mentioned polynomial ambiguity which is dealt with in the same manner.

The divergences in an effective theory have the same properties as those arising in an ordinary renormalizable lagrangian.⁷ For gauge theories there are power divergences associated only with unprotected masses and super-renormalizable couplings, while the logarithmic divergences determine the renormalization group running of the couplings in the theory. Note that working to a fixed order in the loop expansion and ordering the terms in \mathcal{L}_{eff} according to their index (section 3), insures that to this number of loops the renormalization group equations involve only a finite number of couplings.

A consequence of the above discussion is that power divergences cannot be ascribed any phenomenological significance; they are relevant only for the naturality of certain scalar masses. Note also that, as a consequence of gauge invariance, the vector boson masses can be kept naturally below Λ (see section 7.3).

7.2. *A simple example.*

As a simple example of a loop computation I will consider the operator

$$\mathcal{O}_{\phi B} = \frac{1}{2} (\phi^\dagger \phi - \frac{1}{2} v^2) (B_{\mu\nu} B^{\mu\nu}) \quad (7.1)$$

in the decoupling scenario, and study its effects on the Z vector boson vacuum polarization. This calculation is done to illustrate how loop corrections and renormalization are carried out within the effective-lagrangian formalism. For more physically relevant loop calculations see Refs. 71, 94, 72.

Since this calculation involves a single insertion of a dimension six operator the results of section 3 insure that all divergences can be absorbed in renormalizing the terms of dimension ≤ 6 in \mathcal{L}_{eff} .

(a) (b)

Figure 1. Radiative corrections to the Z vacuum polarization
generated by an effective vertex — denoted by the black
dot (wavy lines: Z vector boson, dashed lines: Higgs).

Expanding the dimension six operator yields

$$\mathcal{O}_{\phi B} = \frac{1}{2} s_W^2 Z_{\mu\nu}^2 \left(\frac{1}{2} H^2 + vH \right) + \dots, \quad (7.2)$$

where $v \simeq 246$ GeV and the dots indicate terms that do not contribute to the calculation at hand (to be done in the unitary gauge). The lagrangian I will consider is

$$\mathcal{L}_{\text{St.Model}} + \frac{\alpha_{\phi B} g'^2}{\Lambda^2} \mathcal{O}_{\phi B}. \quad (7.3)$$

The relevant graphs are presented in figure 1. The first diagrams give, in dimensional regularization,

$$\text{Fig. 1a} = \frac{i\alpha_{\phi B} (g' s_W)^2 m_H^2}{8\pi^2 \Lambda^2} (C_{UV} + 1 - \ln m_H^2) (p^2 g^{\alpha\beta} - p^\alpha p^\beta), \quad (7.4)$$

where $C_{UV} = 2/(4-n) - \gamma_E + \ln 4\pi$ (γ_E = Euler's constant and n is the dimension of space time).

The second graphs give

$$\text{Fig. 1b} = -\frac{i\alpha_{\phi B} (g' s_W)^2 m_Z^2}{4\pi^2 \Lambda^2} (C_{UV} - \mathcal{I}) (p^2 g^{\alpha\beta} - p^\alpha p^\beta). \quad (7.5)$$

The integral \mathcal{I} is defined by

$$\mathcal{I} = 2 \int_0^1 dx \, x \ln [x m_H^2 + (1-x) m_Z^2 - x(1-x) p^2]. \quad (7.6)$$

Therefore the contribution to the Z vacuum polarization, which I denote by $\delta\Pi_Z^{\alpha\beta}$ is transverse, with $\delta\Pi_Z^{\alpha\beta} = \delta\Pi_Z(g^{\alpha\beta} - p^\alpha p^\beta/p^2)$, where

$$\delta\Pi_Z = \frac{\alpha_{\phi B} (g' s_W)^2 (m_H^2 - 2m_Z^2) p^2}{8\pi^2 \Lambda^2} C_{UV} + \text{finite}. \quad (7.7)$$

This divergence can be absorbed in a redefinition of $\alpha_{\phi B}$ itself: replacing (the dots indicate higher loop corrections)

$$\alpha_{\phi B} \rightarrow \alpha_{\phi B}^{(0)} = \alpha_{\phi B} \left[1 + \frac{2m_Z^2 - m_H^2}{8\pi^2 v^2} C_{UV} + \dots \right] \quad (7.8)$$

in (7.3) cancels the divergence in (7.6). With this (\overline{MS}) renormalization prescription, and taking

for simplicity the the limit where $m_H^2 \gg m_Z^2, p^2$, I obtain

$$\delta\Pi_Z^{(\text{ren})} \simeq \frac{\alpha_{\phi B}(g's_W)^2 m_H^2 p^2}{8\pi^2 \Lambda^2} \left[1 - \ln \frac{m_H^2}{\kappa^2} + \frac{2m_Z^2}{m_H^2} \left(\ln \frac{m_H^2}{\kappa^2} - \frac{1}{2} \right) \right], \quad (7.9)$$

where κ is the renormalization scale.

It is now natural to ask whether all divergences generated by $\mathcal{O}_{\phi B}$ can be absorbed in redefining its coefficient. This is in fact not the case; for example the four Higgs Green's function gets a divergent contribution corresponding to the interaction $H^2 \square H^2$. The full set of divergences will not be described here, I merely point out that they can all be absorbed in the coefficients of $\mathcal{O}_{\phi B}$ or of the operators $(\phi^\dagger \phi - v^2/2)|D_\mu \phi|^2$, $(\phi^\dagger \phi) \square (\phi^\dagger \phi)$ and $B^{\mu\nu} (D_\mu \phi)^\dagger (D_\nu \phi)$.

This example illustrates the claims made above: the radiative corrections with an effective lagrangian can be carried out in the same manner as for “usual” lagrangians. The divergences encountered correspond to local operators and therefore can be absorbed in a renormalization of the existing effective lagrangian coefficients. In this case the operators which are renormalized have the same (or lower) index as $\mathcal{O}_{\phi B}$ since the calculation is done in the decoupling scenario, see section 3. From the expression for the counter-terms, the relevant beta functions can be derived and the running effective couplings can be obtained. Thus the whole renormalization program²⁷ can be carried over into the effective-lagrangian formalism without conceptual difficulty.

Similar calculations can be done in the non-decoupling scenario, see, for example, Refs. 36, 37, 70, 71. In particular the first three references explicitly demonstrate that the divergent terms satisfy the symmetries of the model and hence can be absorbed in a redefinition of the effective lagrangian coefficients.

When considering quantitative predictions it should not be forgotten that many processes receive tree-level contributions from \mathcal{L}_{eff} . Tree level contributions must occur whenever the loop contributions diverge, otherwise these divergences would not be absorbable in the redefinition of the coefficients of the effective lagrangian. The converse is not true: some loops may be

accidentally finite, such as the \mathcal{O}_W (see (5.2)) contributions to the anomalous magnetic moment of the muon,^{73,71} and still there can be a tree-level operator. This point is quantitatively important since loop contributions such as (7.9) are suppressed by coupling constants and factors of $1/16\pi^2$. See section 8.2 for an example.

In some cases, however, the tree-level operator contributing to a certain process is forbidden by a symmetry. It then follows that the loop contributions must be finite, even if they involve higher dimension operators. An example of this situation is the case where one looks at effective lagrangian corrections for the two-Higgs-doublet version of the Standard Model⁷⁴ and considers the process $a_0 \rightarrow \gamma\gamma$, where a_0 is the CP-odd scalar of this model.⁷⁵ If the usual discrete symmetry is imposed to avoid flavour changing neutral currents,⁷⁴ then it is easy to see that there are no tree level contributions from any operator of dimension six (though they do occur in higher dimensional operators). It follows that the calculation of this reaction including operators of dimension six and lower will be finite, which is verified explicitly.

7.3. *Radiative corrections and gauge invariance.*

Gauge invariance has been shown to be a very useful principle, but it is conceivable that this happens to be a low-energy effective symmetry which is not respected by the heavy physics.^l If true this would imply that there would be deviations from the gauge invariant couplings generated by the heavy physics; these coprolites could then produce measurable effects at various experiments. I will argue that this is very unlikely based on an argument originally put forth by Veltman.⁶¹ The argument is based on the observation that only when gauge bosons have Yang-Mills type interactions will the so-called “delicate gauge cancellations” be present. Any deviation from this type of interaction will ruin this balance with disastrous consequences.

^l Note that by “heavy” I tacitly understand that the scale Λ is at most a few tens of TeV, the case where Λ is the Plank scale⁷⁶ is not studied here.

Consider a model where the vector bosons have a mass of order M and consider the radiative corrections to the triple vector boson couplings and to the fermion-anti fermion-gauge boson couplings. Schematically a triple vector boson coupling has the form gW^3p , where W denotes the vector boson field, g the corresponding gauge coupling constant, and p a generic momentum, produced by a derivative which must be present in such a vertex. The fermion coupling will be of the form $g_f\bar{\psi} \not{W}\psi$, where g_f denotes the corresponding coupling constant. Since this is not a gauge theory (W couplings are not precisely those of the Yang-Mills type but have small corrections generated by the heavy physics), the only consistent vector boson propagator has the form $(g_{\mu\nu} - p_\mu p_\nu/M^2)/(p^2 - M^2 + i\epsilon)$.

Graph 2.a below gives a correction of order $\Lambda^6 g^2/(16\pi^2 M^4)$ to M^2 which should, by naturalness, be $\lesssim M^2$; the largest allowed value of Λ corresponds to $M^2 \sim \Lambda^6 g^2/(16\pi^2 M^4)$. This implies

$$\frac{g}{4\pi} \sim \frac{M^3}{\Lambda^3}. \quad (7.10)$$

Similarly, graph b in Fig. 2 gives a correction to the coupling g_f of order $g_f^3 \Lambda^2/(16\pi^2 M^2)$, which can at most be $\sim g_f$; this requires

$$\frac{g_f}{4\pi} \sim \frac{M}{\Lambda}. \quad (7.11)$$

For a theory which also satisfies $g \sim g_f$ (as usually imposed) the above naturality arguments imply $M \sim \Lambda$ and $g \sim g_f \sim 4\pi$. Therefore the theory is strongly coupled. More important, however, is the fact that radiative corrections will shift M to the cutoff: the vector bosons do not belong to the low energy theory at all. Although this is far from being a rigorous argument it does show that a very careful cancellation of divergences is required if the masses of the vector bosons are to be kept naturally light.

(a)

(b)

Figure 2. Radiative corrections for a non-gauge theory (curly

lines: vector bosons, solid lines: fermions).

Thus the assumption that whatever generates the Standard Model as a low energy effective lagrangian can violate gauge invariance above a certain scale and also leave some gauge variant remnants whose effects can be of relevance to any present or near-future experiments, is inconsistent with the requirements that the W mass is significantly below this scale, and with the measurements $g_f, g \sim 0.65$.

These arguments in favor of explicit gauge invariance apparently contradict the previous statements to the effect that any theory can be rendered gauge invariant (sect. 2). To understand how these points are reconciled, recall first that possible contradiction arises only in the non-linear realization of the symmetry (else there are no auxiliary unitary fields present and the classical lagrangian must be manifestly gauge invariant).

As mentioned in section 4.1 the corrections to M (obtained by setting $C = 2$, $B = 0$, $A' = 2$ in (4.7)), are of order $g\Lambda/(4\pi)$. Moreover $g_f = g$, which follows from the fact that gauge fields appear always inside a covariant derivative. These results imply that naturality is perfectly consistent for a theory which has been rendered gauge invariant using the Stuckelberg trick, provided the gauge boson mass is related to the cutoff in the above manner; identifying $\Lambda = 4\pi v$ yields the familiar result $M \sim gv$. The difference between this calculation and the previous one

lies in the intrinsic properties of gauge-invariant theories: only in these models the vector-boson propagators have a reasonable high-energy behaviour, and this leads to much milder constraints. In contrast, when gauge invariance is not present, the only consistent vector boson propagators do not drop off at large momenta, and this leads to unacceptably large corrections to the vector boson masses; the theory, moreover, must also be strongly coupled. Both these results are unacceptable for the electroweak sector.

An important conclusion to be drawn from these arguments is that the fact that we observe the W and Z bosons with masses significantly below the Fermi scale gives a very strong argument for imposing gauge invariance in the lagrangian (be it via the Stuckelberg trick or directly).

7.4. *Anomalies.*

The presence of new fermion-gauge boson couplings in \mathcal{L}_{eff} raises the possibility that new anomalies are generated. This is most easily discussed by using a high derivative regularization.⁷⁷ The basic idea is to replace

$$\bar{\psi} i \not{D} \psi \rightarrow \bar{\psi} \left\{ i \not{D} \left[1 + (-D^2/M^2)^n \right] \right\} \psi, \quad (7.12)$$

so that the propagator drops off as $1/p^{2n+1}$ at large momentum p . Using this regularization method simple power counting shows that all graphs but the usual triangle (with the minimal substitution fermion-gauge boson vertices) ones are well defined whenever n is sufficiently large. This implies that the axial currents would be well defined in the regulated theory, were it not for these graphs, which correspond to the usual anomalous diagrams. It follows that there are no new anomalies generated by the higher dimensional operators.

8. Applications.

There has been recently a surge of papers applying the effective lagrangian parametrization to various processes covering a wide variety of subjects, from the recent observation of the $B \rightarrow K^* \gamma$ decay to various reactions in $\gamma\gamma$ (back scattered laser) colliders. It is impossible to review all of these papers in any detail and so I am forced to make a selection of various cases which are illustrative; I will, however, mention related results when appropriate.

The results will be organized as follows. Given an experiment I will present the operators whose effects have been studied together with the bounds on their coefficients (either real bounds from current data or expected sensitivity for future experiments). In the discussion below the reader will often find the phrase “the sensitivity limits are” followed by an equation of the type $|quantity| < bound$. This should be understood to mean that the experiment in question will be insensitive to “*quantity*” if its values happen to lie in the interval $(-bound, +bound)$.

Sections 8.1 — 8.11 deal with specific experiments. the results are summarized in the tables of section 8.12. The reader not interested in a specific case may proceed to that section directly.

For the most of this section the coefficients will be assumed to take their natural sizes (see section 4) which entails, aside from possible positive or negative powers of 4π , the presence of gauge coupling constants whenever a gauge field is present. In this respect there are two different situations which lead, in general, to different natural values for the coefficients. In the non-decoupling scenario the relevant estimates are obtained from (4.7). For the decoupling scenario, following the discussion of section 4.2, I will assume that the underlying physics is weakly coupled, hence the coefficients are obtained by determining whether an operator is tree or loop generated (section 4.2); the coupling constants will be assumed to be ~ 1 .

When considering the contributions from \mathcal{L}_{eff} to an observable it is often found that many terms contribute. The modification to the Standard Model prediction will be then proportional

to a linear combination of the coefficients of the contributing operators. I will assume that there are no (significant) cancellations between these coefficients in order to derive bounds on them. Alternatively one can say that this linear combination defines a unique operator which contributes to the observable of interest, and that the coefficient of this operator can be estimated following the arguments of section 4.

In the decoupling scenario the operators for which I will present limits are

$$\begin{aligned}
\mathcal{L}_{lW} &= \frac{1}{\Lambda^2} g \alpha_{lW} (\bar{\ell}_l \sigma^{\mu\nu} \sigma_I l_R) \phi W_{\mu\nu}^I; \\
\mathcal{L}_{lB} &= \frac{1}{\Lambda^2} g' \alpha_{lB} (\bar{\ell}_l \sigma^{\mu\nu} l_R) \phi B_{\mu\nu}; \\
\mathcal{L}_W &= \frac{1}{\Lambda^2} g^3 \alpha_W \epsilon_{IJK} W_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K; \\
\mathcal{L}_{\tilde{W}} &= \frac{1}{\Lambda^2} g^3 \alpha_{\tilde{W}} \epsilon_{IJK} \tilde{W}_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K; \\
\mathcal{L}_{WB} &= \frac{1}{\Lambda^2} g g' \alpha_{WB} \phi^\dagger \sigma_I \phi W_{\mu\nu}^I B_{\mu\nu}; \\
\mathcal{L}_{\tilde{W}B} &= \frac{1}{\Lambda^2} g g' \alpha_{\tilde{W}B} \phi^\dagger \sigma_I \phi \tilde{W}_{\mu\nu}^I B_{\mu\nu}; \\
\mathcal{L}_{4\psi;vec} &= \frac{1}{\Lambda^2} \alpha_{4\psi;vec} \sum_{f=q,\ell_l} (\bar{q} \gamma^\mu q) (\bar{\ell}_l \gamma_\mu \ell_l); \\
\mathcal{L}_{4\psi;sc} &= \frac{1}{\Lambda^2} \alpha_{4\psi;sc} [(\bar{\ell} e) \epsilon (\bar{q} u) + 2 (\bar{\ell} u) \epsilon (\bar{q} e)]; \\
\mathcal{L}_{\phi f}^{(3)} &= \frac{1}{\Lambda^2} \alpha_{\phi f}^{(3)} \left[\sum_l (\bar{\ell} \gamma^\mu \sigma_I \ell) (\phi^\dagger \sigma_I D_\mu \phi) + \sum_q (\bar{q} \gamma^\mu \sigma_I q) (\phi^\dagger \sigma_I D_\mu \phi) \right]; \\
\mathcal{L}_{\phi W} &= \frac{1}{\Lambda^2} \frac{g^2 \alpha_{\phi W}}{2} (\phi^\dagger \phi) (W_{\mu\nu}^I)^2; \\
\mathcal{L}_{\phi \tilde{G}} &= \frac{1}{\Lambda^2} \frac{g_s^2 \alpha_{\phi \tilde{G}}}{2} (\phi^\dagger \phi) \tilde{G}_{\mu\nu}^A G^{A\mu\nu}; \\
\mathcal{L}_\phi^{(1)} &= \frac{1}{\Lambda^2} \alpha_\phi^{(1)} (\phi^\dagger \phi) (D_\mu \phi^\dagger D^\mu \phi); \quad \mathcal{L}_\phi^{(3)} = \frac{1}{\Lambda^2} \alpha_\phi^{(3)} |\phi^\dagger D_\mu \phi|^2;
\end{aligned} \tag{8.1}$$

(the notation is given in Eqs. (1.2)-(1.4) and the comments following them.). I have assumed that the couplings in $\mathcal{L}_{4\psi;vec}$ are all identical; this is done for simplicity only, for a more detailed analysis see, for example Refs. 54 and 53.

The natural order of magnitude for the above coefficients is

$$\begin{aligned} \alpha_{4\psi;vec,sc}, \alpha_{\phi f}^{(3)}, \alpha_{\phi}^{(1,3)} &\sim 1; \\ \alpha_{lW}, \alpha_{lB}, \alpha_{WB}, \alpha_{\tilde{W}B}, \alpha_{\phi W}, \alpha_{\phi\tilde{G}}, \alpha_W, \alpha_{\tilde{W}} &\sim \frac{1}{16\pi^2}. \end{aligned} \quad (8.2)$$

In the non-decoupling scenario the operators considered are

$$\begin{aligned} \mathcal{L}'_1 &= \frac{1}{4}\beta_1 g^2 v^2 \{ \text{tr} (\sigma_3 U^\dagger D_\mu U) \}; \\ \mathcal{L}_1 &= \frac{1}{4}\alpha_1 g g' B^{\mu\nu} W_{\mu\nu}^I (U \sigma_3 U^\dagger \sigma_I); \\ \mathcal{L}_2 &= i\alpha_2 g' B^{\mu\nu} \text{tr} (\sigma_3 D_\nu U^\dagger D_\mu U); \\ \mathcal{L}_3 &= i\alpha_3 g W_{\mu\nu}^I \text{tr} (\sigma_I D^\nu U^\dagger D^\mu U); \\ \mathcal{L}_4 &= \alpha_4 \{ \text{tr} (D_\nu U^\dagger D_\mu U) \}^2; \\ \mathcal{L}_5 &= \alpha_5 \{ \text{tr} (D^\mu U^\dagger D_\mu U) \}^2; \\ \mathcal{L}_8 &= \frac{1}{16}\alpha_8 g^2 \{ W_{\mu\nu}^I \text{tr} (U^\dagger \sigma_3 U \sigma_I) \}^2 \\ \mathcal{L}_{11} &= \alpha_{11} g \tilde{W}_{\mu\nu}^I [\text{tr} (\sigma_3 U^\dagger D^\mu U)] [\text{tr} (U^\dagger \sigma_I D^\nu U)]; \end{aligned} \quad (8.3)$$

where $D_\mu U = \partial_\mu U - \frac{i}{2}g\sigma_I W_\mu^I U - \frac{i}{2}g'B_\mu U\sigma_3$. The operators $\mathcal{L}_{4\psi;vec,sc}$ also appear in the non-decoupling scenario to the order considered here (index ≤ 2). The estimates from section 4 imply $\alpha_i \sim 1/16\pi^2$. We also would have $\beta_1 \sim 1$ but, as mentioned before, there are extra suppression factors (of unknown origin) which require this constant to be $\lesssim 1\%$ (which coincidentally is of the same order as the α_i). The terms $\mathcal{L}_{4\psi;vec,sc}$ are also present in the non-decoupling scenario to the order we are working; as discussed in section 4 their coefficients will be $\sim 16\pi^2$, that is, these terms are $\propto 1/v^2$. In the following I will assume

$$\alpha_i, \beta_1 \sim \frac{1}{16\pi^2}; \quad \alpha_{4\psi;vec,sc} \sim 16\pi^2. \quad (8.4)$$

Two tables will be given, one where the sensitivity of the various present and future experiments to Λ when the coefficients of \mathcal{L}_{eff} take their natural values, and another where the bounds

on the coefficients are derived when $\Lambda = 1$ TeV. This choice is made for convenience. It must be remembered that in the decoupling scenario when the underlying theory is weakly coupled, there may be contributions arising from tree-level-generated dimension eight operators which can dominate over the ones presented here (section 4.2). The crossover occurs at scales ~ 3 TeV. This can produce enhancements of the order of $(4\pi v/\Lambda)^2 \sim 10$ (when $\Lambda = 1$ TeV) in the expected magnitude of the measured effects. On the other hand there may be small coupling constants that can decrease the contribution by an order of magnitude or more. The results should be interpreted keeping these caveats in mind.

Before proceeding to the predictions a comment on the triple vector boson couplings is needed. The most general Lorentz invariant lagrangian describing such terms (see, for example, Ref. 18)

$$\begin{aligned}
\mathcal{L}_{WWV}/g_{WWV} = & ig_1^V (W_{\mu\nu}^\dagger W^\mu V^\nu - \text{h.c.}) - g_4^V W_\mu^\dagger W_\nu (\partial^\mu V^\nu + \partial^\nu V^\mu) \\
& + i\kappa_V W_\mu^\dagger W_\nu V^{\mu\nu} + i\frac{\lambda_V}{m_W^2} W_{\lambda\mu}^\dagger W^\mu{}_\nu V^{\nu\lambda} \\
& + i\tilde{\kappa}_V W_\mu^\dagger W_\nu \tilde{V}^{\mu\nu} + i\frac{\tilde{\lambda}_V}{m_W^2} W_{\lambda\mu}^\dagger W^\mu{}_\nu \tilde{V}^{\mu\nu} \\
& + g_5^V \epsilon^{\mu\nu\rho\sigma} \left(W_\mu \overleftrightarrow{\partial}_\rho W_\nu \right) V_\sigma
\end{aligned} \tag{8.5}$$

(where terms proportional to $\partial \cdot W$ and $\partial \cdot V$ are ignored). V denotes either the photon or the Z field; in the first case only terms in compliance with electromagnetic gauge invariance are retained. It is assumed (without loss of generality) that $g_{WW\gamma} = -e$, $g_{WWZ} = -e \cot \theta_W$.

In the literature bounds on the couplings κ_V, λ_V , etc. are often found. It is important to remember, however, that the use of a consistent gauge-invariant effective lagrangian expansion imposes severe constraints among these couplings. In the decoupling scenario

$$\begin{aligned}
\lambda_\gamma = \lambda_Z = \frac{6m_W^2 g^2}{\Lambda^2} \alpha_W; \quad \tilde{\lambda}_\gamma = \tilde{\lambda}_Z = \frac{6m_W^2 g^2}{\Lambda^2} \alpha_{\tilde{W}} \\
\kappa_\gamma - 1 = \kappa_Z - 1 = \frac{4m_W^2}{\Lambda^2} \alpha_{WB}; \quad \tilde{\kappa}_\gamma = \tilde{\kappa}_Z = \frac{4m_W^2}{\Lambda^2} \alpha_{\tilde{W}B}.
\end{aligned} \tag{8.6}$$

In the non-decoupling scenario, and in the notation of appendix B, the corresponding rela-

tions for the CP conserving operators are⁴⁵

$$\begin{aligned}
g_1^Z - 1 &= \frac{g^2}{c_{2W}}\beta_1 + \frac{g^2 t_W^2}{c_{2W}}\alpha_1 + \frac{g^2}{c_W^2}\alpha_3; \\
\kappa_Z - 1 &= \frac{g^2}{c_{2W}}\beta_1 + \frac{g^2 t_W^2}{c_{2W}}\alpha_1 + g^2 t_W^2(\alpha_1 - \alpha_2) + g^2(\alpha_3 - \alpha_8 + \alpha_9); \\
\kappa_\gamma - 1 &= g^2(-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9); \\
g_5^Z &= g^2 t_W^2 \alpha_{11}; \\
g_5^\gamma &= g_1^\gamma - 1 = \lambda_Z = \lambda_\gamma = 0.
\end{aligned} \tag{8.7}$$

Some of these constants can be ~ 0.01 if all contributions add constructively. If this is not the case they are ~ 0.003 .

In the following discussion I will continuously refer to these relations. It must be pointed out, however, that (8.6) and (8.7) apply only for the operator basis chosen. If, for example, the operator $B^{\mu\nu} D_\mu \phi^\dagger D_\nu \phi$ is added, its coefficient shifts κ_Z away from κ_γ . This operator is redundant (in the sense of section 5) and hence its addition cannot affect any observable: its effect can be absorbed in a redefinition of the coefficients of the Standard Model parameters and other operators. I will consistently use the basis presented in Ref. 40 for the following analysis.

I will not present all the references to the experimental papers but instead refer the reader to the papers cited for their precise sources.

8.1. *Low energy results.*

In this sub-section I present some of the more spectacular bounds on Λ derived from low energy effects. This is very far from a complete list, the reader is referred to the extensive study in Ref. 40 of the low energy effects of the dimension six operators in the decoupling scenario.

- *Neutron dipole moment.* The term $\mathcal{L}_{\phi\tilde{G}}$ generates a contribution to the CP violating θ parameter, which is $< 10^{-9}$; ⁷⁸ this implies

$$\frac{|\alpha_{\phi\tilde{G}}|}{\Lambda_{\text{TeV}}^2} < 10^{-10}. \tag{8.8}$$

Using (8.2) this implies $\Lambda_{\text{TeV}} > 8000$. This presents a bound on the scale at which processes contributing to the neutron dipole moment occur.

- *$K_L - K_S$ mass difference.* The contributions to this quantity depend on operators which violate flavor conservation; the bounds on Λ should then be identified as constraints on the scale at which flavor-changing neutral currents are generated. In this example only I will include more than one generation of fermions. The terms in \mathcal{L}_{eff} which I will consider is

$$\mathcal{L}_{FCNC} = \frac{\alpha_{FCNC}}{2\Lambda^2} (\bar{q}_1 \gamma_\mu q_2)^2 \quad (8.9)$$

where q_1 denotes the up-down quark doublet and q_2 the charm-strange quark doublet.

The contribution to the neutral kaon mass difference requires

$$\frac{|\alpha_{FCNC}|}{\Lambda_{\text{TeV}}^2} < 7 \times 10^{-7}. \quad (8.10)$$

This limit corresponds to $\Lambda_{\text{TeV}} > 1200$ when $|\alpha_{FCNC}| = 1$.

The above bounds are special in that they are obtained for operators which are associated with certain very much suppressed processes within the Standard Model. Thus the corresponding scales might easily be very different from the Λ appearing in other operators.^m For other operators the bounds obtained in Ref. 40 are comparable or weaker than the ones discussed below (albeit using some operators not included in (8.1)).

8.2. **AGS821.**

The Brookhaven experiment AGS821⁷⁹ is expected to measure $a_\mu = (g_\mu - 2)/2$ for the muon to an accuracy of 4×10^{-10} . This can be used to determine the expected sensitivity to

^m This same comment applies to $\mathcal{L}_{4\psi;sc}$ in (8.1).

the anomalous magnetic moment coupling effective operators⁷¹ $\mathcal{O}_{\mu W}$ and $\mathcal{O}_{\mu B}$ in (8.1). The parameter values to which this experiment will not be sensitive lie in the region

$$\frac{|\alpha_{\mu W} - \alpha_{\mu B}|}{\Lambda_{\text{TeV}}^2} < 1.1 \times 10^{-5}. \quad (8.11)$$

In most models, chiral symmetry implies that the coefficients $\alpha_{\mu W, \mu B}$ are suppressed by the muon Yukawa coupling y_μ . This is suggested naturally by the smallness of the muon mass (but can be avoided in some models, see Ref. 71). Then, assuming no cancellations, the above limits imply, for the decoupling scenario, $\Lambda_{\text{TeV}} > 0.5$. For the non-decoupling scenario the same results hold provided the substitution $\Lambda \rightarrow v \simeq 0.246$ TeV is made. The CERN data⁷⁸ strongly favors the presence of the y_μ suppression factor. In the tables this suppression factor is assumed.

This measurement has also been used to put a limit on the effective couplings among three vector bosons (see Ref. 73 for a review). With the natural value for the coefficients the contributions from the corresponding operators are unobservable unless Λ is a few GeV (in the decoupling scenario). For a full discussion see Ref. 71.

8.3. *CLEO*.

The measurement of the branching ratio $B \rightarrow K^* \gamma$ ⁸⁰ has been used to impose bounds on the effective couplings among triple vector boson vertices. The bounds derived from the CLEO measurement are⁸¹ $-0.13 \lesssim 1 - \kappa_\gamma \lesssim 0.75$, $-2.2 \lesssim \lambda_\gamma \lesssim 0.4$, $|\tilde{\kappa}_\gamma| \lesssim 0.32$, $|\tilde{\lambda}_\gamma| \lesssim 0.93$; when the top mass is $m_{\text{top}} = 150$ GeV. Weaker bounds ($|\lambda_\gamma| \lesssim 10$ when all other constants in (8.5) vanish) were obtained in Ref. 82. This implies, using (8.6),

$$\begin{aligned} -5.1 < \frac{\alpha_{WB}}{\Lambda_{\text{TeV}}^2} < 29.3 & \quad \frac{|\alpha_{\tilde{W}B}|}{\Lambda_{\text{TeV}}^2} < 12.5 \\ -135.6 < \frac{\alpha_W}{\Lambda_{\text{TeV}}^2} < 24.7 & \quad \frac{|\alpha_{\tilde{W}}|}{\Lambda_{\text{TeV}}^2} < 57.3 \end{aligned}$$

In Ref. 83 the effects of \mathcal{L}_{11} in (8.3) on the decays $B_s \rightarrow \mu^+ \mu^-$ were studied. The sensitivity

limit is

$$|\alpha_{11}| \lesssim 1.8; \quad (8.12)$$

similar results are obtained from the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

8.4. **HERA.**

There have been several studies into the bounds on four fermi operators that can be obtained by HERA. The possible four fermi interactions can be understood as being generated via a heavy vector exchange, $\mathcal{L}_{4\psi;vec}$; or a heavy scalar exchange, $\mathcal{L}_{4\psi;sc}$. These are labelled vector and scalar exchange respectively.ⁿ The coefficients for these terms have large natural value (see (8.2)) which implies good sensitivity to Λ .

The vector exchange terms were studied in, for example, Refs. 53, 56 using deep inelastic polarized e^\mp -nucleon scattering. A typical result is⁵⁶

$$\frac{|\alpha_{4\psi;vec}|}{\Lambda_{\text{TeV}}^2} \lesssim 0.4 \quad (8.13)$$

The scalar exchanges have been considered in Refs. 84, 56, 85. These are based on the possibility of using polarized electrons to probe helicity violating interactions. Low energy data⁴⁰ restrict these interactions to the form^o $\mathcal{L}_{4\psi;sc}$ in (8.1). For 70% polarization the sensitivity is to

$$\frac{|\alpha_{4\psi;sc}|}{\Lambda_{\text{TeV}}^2} < 7. \quad (8.14)$$

The lagrangian (8.5) has also been studied for this accelerator. In Ref. 86 the reaction $ep \rightarrow \nu \gamma X$ is used to set the bounds $|\kappa - 1| \lesssim 1.4$; $|\lambda| \lesssim 1.1$ (a similar study,⁸⁷ but for five year's

ⁿ All other possibilities are equivalent via a Fierz transformation.

^o Using a Fierz transform this can be identified with a tensor exchange.

integrated luminosity, improves these limits to $|\kappa - 1| \lesssim 0.3$ and $|\lambda| \lesssim 0.8$). These are too weak to be of interest in the non-decoupling scenario; for the decoupling scenario they imply

$$\frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 68; \quad \frac{|\alpha_{WB}|}{\Lambda_{\text{TeV}}^2} < 55. \quad (8.15)$$

8.5. **LEP1.**

The wealth of experimental data from LEP1 has been used by several authors to impose bounds on various operators.

- *Four fermi interactions.* As in the colliders reviewed above, the four fermion interactions are very sensitive to new physics. For example, the forward-backward asymmetry in $b\bar{b}$ production generates a limit¹¹

$$\frac{|\alpha_{4\psi;vec}|}{\Lambda_{\text{TeV}}^2} < 1.9 \quad (8.16)$$

- *Measurements with LEP data and m_W as inputs.* Another good limit on Λ in the decoupling scenario is obtained in Ref. 6 for the coefficient of the operator $\mathcal{O}_\phi^{(1)}$ in (8.1).^p According to (8.2) the coefficient of such an operator is comparatively large. The bounds obtained are

$$-0.132 < \frac{\alpha_\phi^{(1)}}{\Lambda_{\text{TeV}}^2} < 0.876 \quad (8.17)$$

This implies $\Lambda \gtrsim 1.5$ TeV. Similar bounds were also obtained from leptonic four fermion operator effects. These results were based on the measurement of the Z mass and widths as well as the W mass and the neutrino cross section ratio.

^p The operator actually considered is $\mathcal{O}_\phi^{(1)} - \frac{1}{2} [\partial(\phi^\dagger \phi)]^2$; the second term, however, vanishes in the unitary gauge.

- *Triple vector boson vertices.* In a related investigation the authors of Ref. 88 fit the values of several operators to the data and obtain the bounds $0.7 < \kappa_\gamma < 1.7$ and $|\lambda_\gamma| < 0.6$ which translates into

$$-12 < \frac{\alpha_{WB}}{\Lambda_{\text{TeV}}^2} < 27; \quad \frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 37. \quad (8.18)$$

These authors also consider bounds on the non-decoupling scenario operators (8.3),

$$-0.11 < \beta_1 < 0.02, \quad -0.05 < \alpha_1 < 0.04, \quad |\alpha_8| < 0.04; \quad (8.19)$$

where all the couplings are naturally of order $1/16\pi^2$, see (8.4).

- *τ anomalous moments.* The authors of Ref. 65 use the $Z \rightarrow \tau^+\tau^-$ decay rate together with the CDF measurement of m_W ⁷⁸ and the ratio of the charged to neutral neutrino cross sections to impose bounds on the contributions to a_τ (the anomalous magnetic moment) and d_τ (the anomalous electric dipole moment) generated by $\mathcal{O}_{\tau B}$ in (8.1). To 2σ they find a bound

$$\frac{|\alpha_{\tau B}|}{\Lambda_{\text{TeV}}^2} < 1.1, \quad (8.20)$$

with the natural size for the coupling given in (8.2).

- *Custodial symmetry breaking.* In Ref. 6 bounds are obtained on the coefficient of the operators \mathcal{O}_{WB} which is essentially the oblique S parameter, see (3.6). The experimental results used⁷⁸ were the W mass, the partial Z widths, the leptonic axial Z coupling and the neutrino cross section ratio. The 2σ limit on this coefficient is

$$-0.6 < \frac{\alpha_{WB}}{\Lambda_{\text{TeV}}^2} < 0.7 \quad (8.21)$$

Using (8.2) implies that LEP1 is sensitive to scales up to ~ 100 GeV (which is essentially the value of \sqrt{s} for this accelerator).

In Ref. 89 breaking of the custodial $SU(2)$ symmetry in the form of an effective $j_\mu^3 B^\mu$ coupling (j_μ^I is the fermionic current of weak isospin index I). In terms of the parametrization of Ref. 40 this coupling corresponds to $\mathcal{O}_{\phi f}^{(3)}$ in (8.1). The measurements of m_W , m_Z and the weak mixing angle imply

$$\frac{|\alpha_{\phi f}^{(3)}|}{\Lambda_{\text{TeV}}^2} \lesssim 1 \quad (8.22)$$

This term, like the four-fermion interactions, has a relatively large coefficient and therefore can provide substantial bounds on Λ .

Other investigations into the custodial symmetry breaking involve the ρ parameter to which I now turn.

- *Oblique parameters.* The oblique parameters S , T and U ⁴¹ within the non-decoupling scenario are obtained in Refs. 64, 90, 91. The results, in the notation of appendix B, are⁴⁵

$$S = -16\pi\alpha_1; \quad T = \frac{8\pi}{s_W^2}\beta_1; \quad U = -16\pi\alpha_8 \quad (8.23)$$

and are expected to be of order $1/\pi$. Estimates of these quantities for several models are also provided in these references. These expressions together with the experimental results⁹² $-0.8 < S < 0.18$, $-0.46 < T < 0.22$ and $-1.03 < U < 0.81$ provide the bounds

$$-0.004 < \alpha_1 < 0.016; \quad -0.004 < \beta_1 < 0.002; \quad -0.016 < \alpha_8 < 0.02. \quad (8.24)$$

These limits are already of the same order as the estimates in (8.4), yet one more indication of the excellence of LEP data.

Similar expressions can be obtained in the decoupling scenario, namely

$$S = 32\pi\alpha_{WB}\frac{v^2}{\Lambda^2}; \quad T = -\frac{4\pi}{s_W^2}\alpha_\phi^{(3)}\frac{v^2}{\Lambda^2}; \quad U = 0, \quad (8.25)$$

where only dimension six operators have been kept. An immediate result is that the parameter U provide a clear differentiation between the non-decoupling scenario and the

decoupling scenario provided the operators of dimension eight in the latter case can be ignored (*i.e.* provided $\Lambda \gtrsim 3$ TeV).⁹³ The above limits on S and T translate into

$$-0.13 < \frac{\alpha_{WB}}{\Lambda_{\text{TeV}}^2} < 0.03; \quad -0.06 < \frac{\alpha_{\phi}^{(3)}}{\Lambda_{\text{TeV}}^2} < 0.13, \quad (8.26)$$

which put significant bounds on Λ : $\gtrsim 300$ GeV.

It must be remembered that the expressions (8.25) depend on the basis of operators chosen, if another basis is chosen they are modified (for an example see Ref. 88); all discrepancies will disappear when the results are expressed in terms of observables.

The effects of (8.5) on the oblique parameters were calculated, for example, in Ref. 72. These authors did not impose the constraints (8.6) and so their estimates are relevant for the non-decoupling scenario only. A typical result is $-1.4 < \kappa_Z - 1 < 0.4$ which is too loose to generate significant information.

- *Radiative effects.* The effects of the operator \mathcal{O}_W are also considered in Ref. 6. They affect the measured observables only through radiative corrections (this is true for all “blind directions”, see sections 5 and 7 for a discussion of this type of operators). The induced shift on the photon, Z and W vacuum polarization tensors gives the limit

$$-1.4 < \alpha_W \frac{m_W^2}{\Lambda^2} \ln \frac{\Lambda^2}{m_W^2} < 4.4; \quad (8.27)$$

where the natural size of the coupling is given in (8.2); the derived bounds on α_W are very weak for $\Lambda > m_W$.

In a related calculation the radiative effects of other blind operators (section 5) were studied in Ref. 94. For example, using the same observables as above, the effective interaction $i\alpha'_{\phi B} g' B^{\mu\nu} D_\mu \phi^\dagger D_\nu \phi$ was bounded with the result $|\alpha'_{\phi B}|/\Lambda_{\text{TeV}}^2 < 6$. As mentioned at the end of section 5, however, this blind direction can be given sight by choosing a different basis. In this case the effects on the ρ parameter (for example) yield better limits and with much less effort.

8.6. *Tevatron.*

The best sensitivity to Λ for the Fermilab Tevatron is obtained from the four-fermion term $\mathcal{L}_{4\psi;vec}$ in (8.1). As an example, in Ref. 95 the sensitivity limit derived is

$$\frac{|\alpha_{4\psi;vec}|}{\Lambda_{\text{TeV}}^2} < 3 \quad (8.28)$$

The investigations concerning the sensitivity of the Fermilab Tevatron to (8.5) have been published in Refs. 96,97. The predicted results depend, of course, on the luminosity available. For example, looking at WZ production with only leptonic decays for the vector bosons gives an expected bound (from CDF) of $|\lambda_Z| < 1.7$ for an integrated luminosity of 4.7/pb. This is improved to $|\lambda_Z| < 0.4$, $|\kappa_Z - 1| < 2$ for 100/pb. For W^+W^- production, with only leptonic W decays selected as the final state gives the bound $|\lambda| < 1$, $|\kappa - 1| < 1.3$ with 4.7/pb of integrated luminosity. Finally, using CDF and D0 data for $W\gamma$ production at 95% confidence level, $|\lambda_\gamma| < 0.31$, $|\kappa_\gamma - 1| < 1.15$. Summarizing, the sensitivity of the Tevatron to (8.5) is determined by $|\lambda| \lesssim 0.4$, $|\kappa_Z - 1| \lesssim 1.5$. This is not very restrictive within the non-decoupling scenario; for the decoupling scenario they correspond to

$$\frac{|\alpha_{WB}|}{\Lambda_{\text{TeV}}^2} < 59; \quad \frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 25; \quad (\text{for } \sim 100/\text{pb}). \quad (8.29)$$

These are, as mentioned above, predictions. The latest measured limits from D0⁹⁸ using $W\gamma$ production are, for 15/pb, $|\lambda| \lesssim 1.2$, $|\kappa_Z - 1| \lesssim 2.6$ corresponding to

$$\frac{|\alpha_{WB}|}{\Lambda_{\text{TeV}}^2} < 102; \quad \frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 74; \quad (\text{for } \sim 15/\text{pb}) \quad (8.30)$$

The natural size for the coefficients is given in (8.2).

8.7. **LEP2.**

There have been many predictions as to the sensitivity of LEP2 to various operator coefficients. The CM energy is assumed to be 190 GeV and the luminosity 500/pb.

The bounds on the four-fermion interaction coefficients are, for the leptonic final states⁵⁷

$$\frac{|\alpha_{4\psi;vec}|}{\Lambda_{\text{TeV}}^2} < 0.13. \quad (8.31)$$

The bounds on (8.5) are derived from differential cross section for the process $e^+e^- \rightarrow W^+W^-$.^{99,18,96} The sensitivity limit of LEP2 is given by $|\kappa_V - 1|, |\lambda_V|, |\tilde{\kappa}_V|, |\tilde{\lambda}_V| \lesssim 0.5$. These are too large for the non-decoupling scenario; in the decoupling scenario they imply the constraints

$$\frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2}, \frac{|\alpha_{\tilde{W}}|}{\Lambda_{\text{TeV}}^2} < 31; \quad \frac{|\alpha_{WB}|}{\Lambda_{\text{TeV}}^2}, \frac{|\alpha_{\tilde{W}B}|}{\Lambda_{\text{TeV}}^2} < 20. \quad (8.32)$$

These bounds are derived assuming $\kappa_\gamma = \kappa_Z$ and $\lambda_\gamma = \lambda_Z$ which follows from the triple vector boson interactions derived from decoupling scenario \mathcal{L}_{eff} (see appendix A); the limits (8.32) are considerably weakened if this assumption is relaxed.⁹⁹

The authors of Ref. 89 have also investigated the $j_\mu^3 B^\mu$ couplings in LEP2 (see the subsection on LEP1). Assuming that the top mass is found at the Tevatron, the expected sensitivity becomes

$$\frac{|\alpha_{\phi f}^{(3)}|}{\Lambda_{\text{TeV}}^2} \lesssim 0.06 \quad (8.33)$$

which improves the sensitivity to Λ by a factor of four, to 2.5 TeV in the weak coupling case, and to 10 TeV in the strong coupling case.

The non-decoupling scenario has also been studied for LEP2. Reference 100 considers the effects of the terms $\mathcal{L}_{1,2,3}$ of (8.3) (assuming $\alpha_2 = \alpha_3$) on W^+W^- production. No significant deviations from the Standard Model were found even if the α were one order of magnitude larger

than the estimates in Eq. (8.4). This is true irrespective of the polarization of initial and/or final states. In reference 101 the limits on the α are determined, the results are

$$-1.6 < \alpha_2 < 4.8; \quad -0.33 < \alpha_3 < 0.21 \quad (8.34).$$

In reference 83 the effects of the P and C violating (but CP conserving) term \mathcal{L}_{11} in (8.3) were studied for the cross section and forward-backward asymmetry of W pair production in polarized electron positron scattering. Right-handed electron cross section are found to be much more sensitive to α_{11} than that for left-handed electrons. Still the required degree of polarization must be exceedingly (probably unrealistically) accurate: $>99\%$. Even in this case the sensitivity limits are $|\alpha_{11}| < 0.2$.

In reference 24 the phenomenological effects of a modified low energy theory are studied. The light particles are assumed to be the usual Standard Model excitations together with a neutral heavy vector boson Z' , an $SU(2)_L$ scalar singlet χ , and three right handed neutrinos. These particles are arranged in an $SU(2) \times U(1) \times U(1)$ gauge theory for which the effective operators are studied. As mentioned in section 2 the number of effective operators is much larger than the ones presented in Ref. 40 due to the increased particle content. Significant effects on, W pair production were found only when the Z' was relatively light ($\lesssim 300$ GeV), when the couplings of the effective operators were enhanced by three orders of magnitude above their natural size. This is true even when the scale for physics beyond the Z' is also quite light, of the order of 350 GeV.

8.8. **LHC.**

- *Four-fermi operators.* A thorough analysis for pp colliders can be found in Ref. 54; the best bounds found were for dilepton production the resulting sensitivity limit being

$$\frac{|\alpha_{4\psi;vec}|}{\Lambda_{\text{TeV}}^2} < 0.07. \quad (8.35)$$

For the decoupling scenario, using (8.2), this gives $\Lambda \gtrsim 3.8$ TeV.

- *Triple vector boson couplings.* In references 101,102 the limits for (8.5) stemming from WZ and $W\gamma$ production are studied. With a luminosity of 10/fb it is found that

$$-0.96 < \alpha_2 < 0.93; \quad -0.07 < \alpha_3 < 0.04, \quad (8.36)$$

which are slightly better than those obtained in Refs. 14, 102. These limits are about one order of magnitude larger than the natural values for these coefficients.

In reference 103 $W\gamma$ +jet production was considered obtaining the limits $-0.04 < \kappa_\gamma - 1 < 0.02$ and $|\lambda_\gamma| < 0.04$. These are about one order of magnitude above the estimates (8.4). For the decoupling scenario they correspond to

$$-1.6 < \frac{\alpha_{WB}}{\Lambda_{\text{TeV}}^2} < 0.8; \quad \frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 2.5. \quad (8.37)$$

In reference 51 the limits expected at LHC for the coefficients of the terms \mathcal{L}_W and $\mathcal{L}_{\phi W}$ are studied within the decoupling scenario. The rationale behind this choice is the assumption that the underlying physics preserves the custodial symmetry. The reaction studied is the production of transverse vector bosons (which in contrasts to the usual non-decoupling scenario scenario¹⁴ is enhanced with respect to the longitudinal vector boson production). They obtain

$$\frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 3.6; \quad \frac{|\alpha_{\phi W}|}{\Lambda_{\text{TeV}}^2} < 7.8. \quad (8.38)$$

The natural magnitude of these couplings are given in (8.2).

8.9. ***LEP×LHC.***

The effects of (8.5) in this accelerator have been considered in Ref. 86 for $\nu\gamma$ production, and in Ref. 87 for lepton–vector-boson production. The assumed luminosities vary from 500/pb

to 5000/pb and both LEP1-LHC and LEP2-LHC are considered. The sensitivity limits are all of the same order of magnitude, namely, $|\kappa - 1| \lesssim 0.3$; $|\lambda| \lesssim 0.2$ which translates into

$$\frac{|\alpha_{WB}|}{\Lambda_{\text{TeV}}^2}, \frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 12 \quad (8.39)$$

assuming the same couplings for the Z and γ cases. The bounds are too weak to be of interest in the non-decoupling scenario.

8.10. **NLC.**

Various versions of this future collider have been considered in the literature. Two popular choices correspond to energies of 0.5 and 1 TeV with luminosities of 10/fb and 44/fb respectively; these will be referred to as the 0.5 TeV and 1 TeV options. The laser option¹⁰⁴ has also been investigated for the $e\gamma$ and $\gamma\gamma$ initial states.

First I consider the effects of the four fermion interactions. These can be gleaned, for example, from studies of sensitivity limits of new gauge bosons at the NLC; from Ref. 55 I obtain

$$\frac{|\alpha_{4\psi;vec}|}{\Lambda_{\text{TeV}}^2} < 0.04 \quad (8.40)$$

corresponding to $\Lambda_{\text{TeV}} > 5$ when $|\alpha_{4\psi;vec}| = 1$.

For the 0.5 TeV option, deviations from the Standard Model results for the reactions $e^+e^- \rightarrow W^+W^-$ and $e^+e^- \rightarrow W^+W^-V$ ($V = Z, \gamma$) generate the limit^{101,105} $|\kappa - 1| \lesssim 5 \times 10^{-3}$, which is improved by a factor of five in the 1 TeV option.¹⁰⁵ These expectations are about a factor of four better than those derived in Ref. 107; this last reference also estimates a sensitivity of $|\lambda| \leq 0.03$, which corresponds to $|\alpha_W| < 2\Lambda_{\text{TeV}}^2$. For twice the luminosity the authors of Ref. 108 estimate $|\kappa - 1| \lesssim 0.05$. These discrepancies arise from the various observables considered as well as from the assumptions regarding the observability of the deviations from the Standard Model. In this

review I will follow the analysis of Ref. 106 which predicts a sensitivity limit of $|\lambda|, |\kappa - 1| < 0.01$, which translates into

$$\frac{|\alpha_{WB}|}{\Lambda_{\text{TeV}}^2} < 0.4; \quad \frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 0.6, \quad (8.41)$$

for the 0.5 TeV option.

The non-decoupling scenario coefficients have also been bounded¹⁰¹ with the results, for the 0.5 TeV option,

$$\begin{aligned} -0.05 < \alpha_2 < 0.4, \quad -0.04 < \alpha_3 < 0.02, \\ |\alpha_4| < 0.7, \quad -0.6 < \alpha_5 < 0.5. \end{aligned} \quad (8.42)$$

For the 1 TeV option the limits become

$$-0.03 < \alpha_2 < 0.1; \quad |\alpha_3| < 0.01. \quad (8.43)$$

These are already in the interesting range of a few percent. For higher energy colliders the sensitivity is improved.¹⁰⁹ Note however that for a CM energy of 1 TeV, the corrections from higher index operators will be $\sim (1 \text{ TeV})/(4\pi v) \sim 33\%$

A bound on α_{11} using W pair production for polarized beams is derived in Ref. 83; the authors consider the total cross section and the forward-backward asymmetry. The sensitivity limit in, for example, the 1 TeV option are $|\alpha_{11}| \lesssim 0.01$ assuming near perfect polarization ($>99\%$).

In reference 107 the process $e\gamma \rightarrow W\nu$ is used (in the 0.5 TeV option) to obtain the sensitivity bounds $-0.07 < \lambda < 0.05$, $-0.13 < \kappa - 1 < 0.07$ which correspond to

$$-4.3 < \frac{\alpha_W}{\Lambda_{\text{TeV}}^2} < 3.1; \quad -5.1 < \frac{\alpha_{WB}}{\Lambda_{\text{TeV}}^2} < 2.7. \quad (8.44)$$

A related investigation⁸³ considered the effects on the reaction $e\gamma \rightarrow ZW\nu$ generated by the term \mathcal{L}_{11} in (8.3). Significant deviations from the Standard Model are found for $\alpha_{11} = 0.2$ though no sensitivity limit is presented.

Finally the reaction $\gamma\gamma \rightarrow W^+W^-$ is studied (in the 0.5 TeV option) deriving the limits $|\lambda| \leq 0.03$; $-0.02 < \kappa - 1 < 0.03$. These are again of interest only in the non-decoupling scenario and correspond to

$$\frac{|\alpha_W|}{\Lambda_{\text{TeV}}^2} < 1.8; \quad -0.8 < \frac{\alpha_{WB}}{\Lambda_{\text{TeV}}^2} < 1.2. \quad (8.45)$$

This reaction was also studied in Ref. 110 but the parametrization used corresponds to a set of dimension eight operators. A consistent interpretation of their results would require a complete list of dimension 8 operators relevant for this process; to derive such a list is a (daunting) task which lies beyond the scope of this review.

8.11. *Other experiments.*

Various authors have studied the decays of Higgs particles and their sensitivity to (8.5). In reference 111 the decay $H \rightarrow \gamma\gamma$ was studied (see also Ref. 6). With the constraint⁶ $|\alpha_{WB}|/\Lambda_{\text{TeV}}^2 < 0.7$ the corresponding width can be up to ten times the Standard Model value so that this will decay will be a sensitive probe into heavy physics. In other publications (see for example Ref. 112) the same process is considered but the treatment of divergences and of gauge invariance is incorrect.

In Ref. 75 the effective lagrangian approach is applied to the case where the low energy fields are the Standard Model plus one additional doublet. This reference studied the decay $a \rightarrow \gamma\gamma$, where a is the CP odd scalar in the model (see Ref. 74 for a clear review). This process occurs via quark loops, but these graphs are suppressed in the limit of large $\tan\beta$; effective operators can in this case dominate the process, making it a good candidate reaction where to look for new physics when (and if) the scalars used in this model are discovered.

8.12. *Tables.*

In this section I will summarize the results in a series of tables. The first two present the limits on Λ for the decoupling scenario obtained using various operators first for present experiments, then for near future ones. The last two tables give the limits on the coefficients α derived from present and future experiments when $\Lambda = 1$ TeV.

Table 1. Limits on Λ in the decoupling scenario derived from existing accelerators. The operators in the left column generate the corresponding limits.

Limits on Λ (in TeV) (natural size couplings)					
Operator	Experiment (present)				
	AGS821	CLEO	HERA	LEP1	Tevatron [†]
$\mathcal{O}_{\mu B}^*$	0.5	—	—	—	—
$\mathcal{O}_{\mu W}^*$	0.5	—	—	—	—
$\mathcal{O}_{\tau B}^*$	—	—	—	0.01	—
\mathcal{O}_W	0.04	0.01	0.01	0.01	0.02
$\mathcal{O}_{\tilde{W}}$	—	0.01	—	—	—
\mathcal{O}_{WB}	0.01	0.01	0.01	0.2	0.01
$\mathcal{O}_{\tilde{W}B}$	—	0.02	—	—	—
$\mathcal{O}_{4\psi;vec}^{\ddagger}$	—	—	1.6	0.7	1.7
$\mathcal{O}_{4\psi;sc}^{\ddagger}$	—	—	0.4	—	—
$\mathcal{O}_{\phi f}^{(3)}$	—	—	—	1.0	—
$\mathcal{O}_{\phi}^{(1)}$	—	—	—	1.1	—
$\mathcal{O}_{\phi}^{(3)}$	—	—	—	2.8	—

* (Yukawa coupling included in coefficient)

[‡] ($|\alpha_{4\psi;vec,sc}| = 1$)

[†] (100/pb)

In table 1 I present the limits on Λ derived from current experimental data when the coefficients have their natural magnitudes. As mentioned previously, these estimates have several caveats that allow situations where the sensitivity is reduced (small coupling constants or suppression due to unknown symmetries), or enhanced (low lying resonances).

Table 2. Limits on Λ in the decoupling scenario expected from

future accelerators. The operators in the left column

generate the corresponding limits.

Limits on Λ (in TeV) (natural size couplings)				
Operator	Experiment (future)			
	LEP2	LHC	LEP \times LHC	NLC*
\mathcal{O}_W	0.01	0.05	0.02	0.10
$\mathcal{O}_{\tilde{W}}$	0.01	—	—	—
\mathcal{O}_{WB}	0.02	0.06	0.02	0.13
$\mathcal{O}_{\tilde{W}B}$	0.02	—	—	—
$\mathcal{O}_{4\psi;vec}^\dagger$	2.8	3.8	—	5
$\mathcal{O}_{\phi f}^{(3)}$	4.1	—	—	—
$\mathcal{O}_{\phi W}$	—	0.03	—	—

* (0.5 TeV option)

$^\dagger (|\alpha_{4\psi;vec}| = 1)$

In many cases the bound on the heavy physics scale lies below \sqrt{s} in collider experiments. Usually, this means that any new physics within grasp of these colliders would have been observed directly. That this is not the case implies that the minimal value of Λ to be deduced from a given accelerator is of order \sqrt{s} , many of the above limits are significantly weaker. Still quite acceptable bounds can be obtained from AGS821 and from LEP1; by choosing observables affected by tree level generated operators, scales of up to a few TeV have been probed.

In table 2 I present the corresponding results for future accelerators.

Again in all but one case the scales probed using effective operators lie below \sqrt{s} and again this implies that if there is any new physics within reach of these accelerators it will be produced directly and will not be inferred indirectly via the low energy effective lagrangian it generates.

The one exception are the expected sensitivity for the four fermion interactions. In the non-decoupling scenario LHC will probe scales of a few TeV in this manner. In the non-decoupling

scenario this bound is extended to 49 TeV; one must remember that the expected scale in this case is $\Lambda \sim 4\pi v \simeq 3$ TeV. It must be remembered that even in this case the determination of new physics effects is far from straightforward.¹³

Table 3. Limits on the effective lagrangian coefficients from

current experiments when $\Lambda=1$ TeV.

Limits on the coefficients ($\Lambda=1$ TeV)					
Coupling	Experiment (present)				
	AGS821	CLEO	HERA	LEP1	Tevatron [†]
$ \alpha_{4\psi;vec} $	—	—	0.4	1.9	3
$ \alpha_{4\psi;sc} $	—	—	7	—	—
$ \alpha_{\phi f}^{(3)} $	—	—	—	1.0	—
$ \alpha_{\phi}^{(1)} $	—	—	—	0.9	—
$ \alpha_{\phi}^{(3)} $	—	—	—	0.13	—
$ \alpha_{\mu B} ^*$	0.03	—	—	—	—
$ \alpha_{\mu W} ^*$	0.03	—	—	—	—
$ \alpha_{\tau B} ^*$	—	—	—	1.1	—
$ \alpha_W $	4	136	68	37	25
$ \alpha_{\tilde{W}} $	—	57.3	—	—	—
$ \alpha_{WB} $	0.2	29	55	0.13	59
$ \alpha_{\tilde{W}B} $	—	12.5	—	—	—
$ \beta_1 $	—	—	—	0.004	—
$ \alpha_1 $	—	—	—	0.02	—
$ \alpha_2 $	—	—	—	—	—
$ \alpha_3 $	—	—	—	—	—
$ \alpha_4 $	—	—	—	—	—
$ \alpha_5 $	—	—	—	—	—
$ \alpha_8 $	—	—	—	0.02	—
$ \alpha_{11} $	—	1.8	—	0.9	—

* (Yukawa coupling included in coefficient)

[†] (100/pb)

Now I turn to the expected and measured sensitivity to the effective lagrangian in future and present experiments. In this case it is of interest to display also the sensitivity for the coefficients in the non-decoupling scenario as well as in the decoupling scenario. I have chosen $\Lambda = 1$ TeV to present these results. The translation to other scale is straightforward by using the formulas presented in the previous sections.

For the decoupling scenario I again emphasize that, since the scale chosen lies below $4\pi v$ the contribution from tree level generated dimension 8 operators could be significant.

Table 4. Limits on the effective lagrangian coefficients expected

from future accelerators when $\Lambda=1$ TeV.

Limits on the coefficients ($\Lambda=1$ TeV)				
Coupling	Experiment (future)			
	LEP2	LHC	LEP \times LHC	NLC*
α_W	31	2.5	12	0.6
$\alpha_{\tilde{W}}$	31	—	—	—
α_{WB}	20	1.6	12	0.4
$\alpha_{\tilde{W}B}$	20	—	—	—
$\alpha_{4\psi;vec}$	0.13	0.07	—	0.04
$\alpha_{\phi f}^{(3)}$	0.06	—	—	—
$\alpha_{\phi W}$	—	7.8	—	—
α_2	4.8	0.96	—	0.4
α_3	0.3	0.07	—	0.04
α_4	—	—	—	0.7
α_5	—	—	—	0.6

* (0.5 TeV option)

The measured (or shortly expected) limits on the effective lagrangian coefficients are presented in table 3 above. The expected magnitude for these coefficients is $\sim 1/16\pi^2 \simeq 0.006$ except for the first five which are naturally of order one (see (8.2) and (8.4)). The coefficient β_1

is proportional to the oblique T parameter and is suppressed, as was discussed above, for some unknown reason (though it may be understood on the basis that the heavy physics respects the custodial symmetry). The coefficients $\alpha_{2,8}$ are also very well measured being proportional to the oblique S and U parameters. The only other constants which are reasonably well measured are those expected to be ~ 1 , not coincidentally these also provide the only significant limits on Λ .

As can be seen from the above table, in order to derive a significant bound on Λ (say, $\Lambda = 1$ TeV) from, for example, the Tevatron data by using the anomalous W couplings, one would have to assume that these couplings are more than four orders of magnitude above their natural magnitudes.

In table 4 I present the expected sensitivity of future colliders, should new physics be at 1 TeV. As can be seen the expected sensitivity to the various coefficients is very weak except for some non-decoupling scenario couplings, especially α_3 (the same will be true for $\alpha_\phi^{(3)}$). I have not included the couplings corresponding to the oblique parameters $\alpha_{2,8}$ and β_1 in this study, but the experimental precision will certainly improve for these measurements, to the point that we must either see some deviations from the Standard Model or else postulate some mechanism that will hide the heavy physics effects from the oblique parameters.

9. Conclusions.

In the previous sections I have presented a scrutiny of various aspects of effective theories and their applications to the weak interaction phenomenology.

Effective theories are a useful instrument for parametrizing new physics effects in a consistent and process and model independent manner. The formalism generates corrections to the Standard Model contributions to any observable quantity in terms of a series with unknown coefficients which are not expected to be fundamental, but combination of (as yet unknown) new constants of the lagrangian describing the underlying physics. Due to the hierarchy inherent in the effective

theories considered, a finite number of (very high precision) experimental data points would determine the whole of the new physics effects to a certain accuracy; the errors are then also estimated using the formalism.

The consistent application of the effective lagrangian parametrization yields very good limits on the scale of new physics (up to several TeV in some cases). Many experimental constraints are, however, very weak implying only that Λ greater than a few GeV. In obtaining these results it is paramount that the coefficients of the effective lagrangian should take reasonable values, for example, λ_Z in (8.5) *cannot* be $O(1)$ if the approach is to be consistent.

If we adopt the optimistic assumption that, should there be new physics at a scale Λ all cross sections will show drastic changes at this scale, then we can use the known bound on W_R masses from CDF [78] to state that $\Lambda \gtrsim 0.5$ TeV for the decoupling scenario. This, though better than many limits obtained in the previous section, can of course be avoided depending on the type of new physics present. But it still a fact that most limits on Λ to be achieved by present colliders are in this ball park. To improve these results colliders of energy significantly larger than the present ones are required or, alternatively, super-high precision measurements (at the one per mil or 0.1 per mil level) are needed. Of these possibilities only the first will determine the kind of new physics present unambiguously. For example, it would be very difficult, if not impossible, to state that a certain deviation in the anomalous magnetic moment of the muon is due to the presence of an anomalous vector boson coupling and not to a slight miscalculation of the QCD effects [113].

Any calculation of the effects of new physics via an effective lagrangian (as opposed to a specific model calculation) must at least be consistent. The implications of this obvious fact, however, are often ignored or forgotten. Two such examples were detailed above:

- (i) One cannot arbitrarily ignore some of the operators. In particular one cannot include in a calculation loops containing effective operators and forget others which contribute at tree

level, unless specific assumptions regarding the underlying physics are made which imply that the tree level contributions are negligible or absent.

- (ii) The results of any calculation are reliable as long as the contributions of the operators which were ignored are significantly smaller than the contributions from the operators which were retained.

If these facts are ignored one can obtain impressive effects which would, apparently, indicate enormous deviations from the Standard Model. These results would be wrong: they were obtained under circumstances where the effective lagrangian parametrization is invalid. If the calculations are performed with the consistency of the parametrization in mind, then the effects are generally small, though not always unmeasurable. For example, processes affected by four-fermi operators produced by a strongly interacting theory would produce measurable deviations from the Standard Model at the LHC provided Λ lies below ~ 47 TeV.

As discussed in section 4.4 one cannot strongly violate the estimates for the coefficient of \mathcal{L}_{eff} without losing all predictability in the model. Even if the use of the theory in loop calculations is forbidden, the rationale behind the inter-relations among the lagrangian coefficients is lost. For example, instead of understanding the universal lepton couplings to the W as a consequence of gauge invariance and the representations carried by the fermions, it becomes an accidental fact.

When the formalism is applied consistently it remains a powerful and useful tool with which to probe, in a model independent manner, physics about which we as of now know very little. Unfortunately it is clear from the results of the last section, that most experiments are sensitive to only a small number of effective interactions. This necessarily limits our ability to probe new physics, for it is possible to imagine situations in which these operators are suppressed. The extraction of more information remains a difficult and arduous task which necessitates much more data (to improve statistics) or, optimally, higher energies.

Acknowledgements:

It is a pleasure to acknowledge the help of M. Einhorn from whom I learned most of the ideas presented in the paper. I also received much help from C. Arzt and M. Golden. I benefited from many discussion with M.A. Perez, U. Baur, D. London and E. Ma. I would also like to thank the Aspen Center for Physics for its hospitality where part of this work was completed. This work was supported in part by an SSC fellowship, by a UC Mexus fellowship and through funds provided by the Department of Energy.

APPENDIX A

In this appendix I will present, for the decoupling scenario, the contributions to the three vector boson interactions derived from the operators of dimension six in the effective lagrangian. This exercise will be useful since many authors have studied the limits put on these interactions by the current experimental data, as well as the expected sensitivity in future colliders. I will use the notation of Ref. 40.

There are six operators which generate couplings among three vector bosons, they are

$$\begin{aligned}\mathcal{O}_W &= \epsilon_{IJK} W_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K \\ \mathcal{O}_{\tilde{W}} &= \epsilon_{IJK} \tilde{W}_{\mu\nu}^I W_{\nu\rho}^J W_{\rho\mu}^K \\ \mathcal{O}_{\phi W} &= \frac{1}{2} (\phi^\dagger \phi) (W_{\mu\nu}^I)^2 \\ \mathcal{O}_{\phi B} &= \frac{1}{2} (\phi^\dagger \phi) (W_{\mu\nu}^I \tilde{W}_{\mu\nu}^I) \\ \mathcal{O}_{WB} &= \phi^\dagger \sigma^I \phi W_{\mu\nu}^I B_{\mu\nu} \\ \mathcal{O}_{\tilde{W}B} &= \phi^\dagger \sigma^I \phi \tilde{W}_{\mu\nu}^I B_{\mu\nu}\end{aligned}\tag{A.1}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $W_{\mu\nu}^I = \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g\epsilon_{IJK} W_\mu^J W_\nu^K$ in which W_μ^I and B_μ denote the $SU(2)_L$ and $U(1)_Y$ gauge fields respectively; the corresponding gauge couplings are g and g' .

The scalar doublet is denoted by ϕ and g is the $SU(2)_L$ gauge coupling constant. Note that all these operators are gauge invariant.

In the unitary gauge ϕ is replaced by its vacuum expectation value; in this case the effects of $\mathcal{O}_{\phi W}$ and $\mathcal{O}_{\phi \tilde{W}}$ can be absorbed in a redefinition of the Standard Model lagrangian parameters. These operators will not be considered further. The effective lagrangian of interest then becomes

$$\begin{aligned} \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{St.Model}} + \frac{1}{\Lambda^2} [g^3 \alpha_W \mathcal{O}_W + g^3 \alpha_{\tilde{W}} \mathcal{O}_{\tilde{W}} + \\ g' g \alpha_{WB} \mathcal{O}_{WB} + g' g \alpha_{\tilde{W}B} \mathcal{O}_{\tilde{W}B}] . \end{aligned} \quad (\text{A.2})$$

Using the results of section 4.2 I can estimate the natural magnitude for the above coefficients. the results are

$$\alpha_{W, \tilde{W}, WB, \tilde{W}B} \sim \frac{1}{16\pi^2}. \quad (\text{A.3})$$

From (A.2) one can easily derive all triple boson vertices generated by the dimension six operators. All such couplings are of the form $W^+ W^- V$ with $V = Z, \gamma$. The WWZ couplings are

$$\begin{aligned} \frac{\Lambda^2}{e \cot \theta_W} \mathcal{L}_{WWZ} = 6ig^2 W_{\nu\rho}^- W_{\rho\mu}^+ \left(\alpha_W Z_{\mu\nu} + \alpha_{\tilde{W}} \tilde{Z}_{\mu\nu} \right) \\ + 4im_W^2 W_{\mu}^- W_{\nu}^+ \left(\alpha_{WB} Z_{\mu\nu} + \alpha_{\tilde{W}B} \tilde{Z}_{\mu\nu} \right), \end{aligned} \quad (\text{A.4})$$

while the $WW\gamma$ couplings are

$$\begin{aligned} \frac{\Lambda^2}{e} \mathcal{L}_{WW\gamma} = 6ig^2 W_{\nu\rho}^- W_{\rho\mu}^+ \left(\alpha_W Z_{\mu\nu} + \alpha_{\tilde{W}} \tilde{Z}_{\mu\nu} \right) \\ - 4im_W^2 W_{\mu}^- W_{\nu}^+ \left(\alpha_{WB} Z_{\mu\nu} + \alpha_{\tilde{W}B} \tilde{Z}_{\mu\nu} \right). \end{aligned} \quad (\text{A.5})$$

Recall that there are 14 WWZ and $WW\gamma$ lorentz invariant (though not manifestly gauge invariant) couplings in (8.5). These are reduced to four within this formalism. If (8.5) is made gauge invariant along the lines of section 2, then the resulting lagrangian mixes terms of different indices (see section 3 for the definition of the index of an operator). Keeping only the terms of the lowest index again results in a reduction in the number of unknown parameters.

Finally note that the operators \mathcal{O}_W and $\mathcal{O}_{\tilde{W}}$ also generate four-vector boson couplings, and it is easy to see that all such vertices involve at least two W fields. It is also important to note that there are no other sources of four vector boson couplings in the operators of dimension six (these are also the only operators generating couplings among five and six vector bosons). This implies that all couplings among three vector bosons or more depend on four parameters only. If CP violations are assumed to be negligible the number of unknown parameters is halved.

APPENDIX B

For convenience I include the expressions for the operators of index two in the non-decoupling scenario, the results are taken from Ref. 45. The notation used is

$$T = U^\dagger \sigma_3 U; \quad V_\mu = (D_\mu U) U^\dagger; \quad \mathbf{W}_{\mu\nu} = \frac{1}{2} W_{\mu\nu}^I \sigma_I, \quad (\text{B.1})$$

where $D_\mu U = \partial_\mu U + \frac{i}{2} g \sigma_I W_\mu^I U - \frac{i}{2} g' B_\mu U \sigma_3$.

The CP conserving terms are

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{2} \alpha_1 g g' B_{\mu\nu} \text{tr}(T \mathbf{W}^{\mu\nu}), & \mathcal{L}_2 &= \frac{1}{2} i \alpha_2 g' B_{\mu\nu} \text{tr}(T [V^\mu, V^\nu]), \\ \mathcal{L}_3 &= i \alpha_3 g \text{tr}(\mathbf{W}_{\mu\nu} [V^\mu, V^\nu]), & \mathcal{L}_4 &= \alpha_4 \{\text{tr}(V_\mu V_\nu)\}^2, \\ \mathcal{L}_5 &= \alpha_5 \{\text{tr}(V_\mu V^\mu)\}^2, & \mathcal{L}_6 &= \alpha_6 \text{tr}(V_\mu V_\nu) \text{tr}(T V^\mu) \text{tr}(T V^\nu), \\ \mathcal{L}_7 &= \alpha_7 \text{tr}(V_\mu V^\mu) \{\text{tr}(T V^\nu)\}^2, & \mathcal{L}_8 &= \frac{1}{4} \alpha_8 g^2 \{\text{tr}(T \mathbf{W}_{\mu\nu})\}^2, \\ \mathcal{L}_9 &= \frac{1}{2} i \alpha_9 g \text{tr}(T \mathbf{W}_{\mu\nu}) \text{tr}(T [V^\mu, V^\nu]), \\ \mathcal{L}_{10} &= \frac{1}{2} \alpha_{10} \{\text{tr}(T V^\mu) \text{tr}(T V^\nu)\}^2, \\ \mathcal{L}_{11} &= 2 \alpha_{11} g \text{tr}(T V^\mu) \text{tr}(V_\nu \tilde{\mathbf{W}}^{\mu\nu}). \end{aligned}$$

The CP violating operators are

$$\begin{aligned}
\mathcal{L}_{12} &= \alpha_{12} g \operatorname{tr}(T V_\mu) \operatorname{tr}(V_\nu \mathbf{W}^{\mu\nu}), & \mathcal{L}_{13} &= 2\alpha_{13} g g' \tilde{B}^{\mu\nu} \operatorname{tr}(T \mathbf{W}_{\mu\nu}), \\
\mathcal{L}_{14} &= 4i\alpha_{14} g' \tilde{B}^{\mu\nu} \operatorname{tr}(T V_\mu V_\nu), & \mathcal{L}_{15} &= 4i\alpha_{15} g \operatorname{tr}(\tilde{\mathbf{W}}_{\mu\nu} V^\mu V^\nu), \\
\mathcal{L}_{16} &= \alpha_{16} g^2 \operatorname{tr}(T \tilde{\mathbf{W}}_{\mu\nu}) \operatorname{tr}(T \mathbf{W}^{\mu\nu}), & & \\
\mathcal{L}_{17} &= i\alpha_{17} g \operatorname{tr}(T \tilde{\mathbf{W}}_{\mu\nu}) \operatorname{tr}(T V^\mu V^\nu), & & \\
\mathcal{L}_{18} &= 2\alpha_{18} g'^2 \tilde{B}^{\mu\nu} B_{\mu\nu}, & \mathcal{L}_{19} &= \alpha_{19} g^2 \operatorname{tr}(T \tilde{\mathbf{W}}_{\mu\nu}) \operatorname{tr}(T \mathbf{W}^{\mu\nu}),
\end{aligned} \tag{B.2}$$

where $\tilde{B}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} B^{\rho\sigma}$ and similarly for $\tilde{\mathbf{W}}$.

Finally the operators containing two fermions and index zero, and which do not correspond to a kinetic or mass terms are

$$\bar{\ell}_l U \sigma_3 U^\dagger \ell; \quad \bar{\ell} U^\dagger (\not{D} U) \ell; \quad \operatorname{tr}(T V_\mu) \bar{l}_R \gamma_\mu l_R; \tag{B.3}$$

plus its counterparts for q , u_R and d_R .

APPENDIX C

In this appendix a simple example will be provided to illustrate the consequences of having a large number of particles in a loop in order to offset the loop suppression factors described in section 4.4.

The example consists of a simple two dimensional model of N heavy fermions interacting with a light scalar field θ . The lagrangian is

$$\mathcal{L} = \frac{1}{2} (\partial\theta)^2 + \sum_{a=1}^N \bar{\psi}_a i \mathcal{D} \psi_a; \quad \mathcal{D} = \not{\partial} + \frac{ig}{2} (\partial_\mu \theta) \gamma^\mu - im e^{ig\theta \gamma_5}. \tag{C.1}$$

This is invariant under $\theta \rightarrow \theta - \alpha$, $\psi \rightarrow \exp(i\alpha g \gamma_5/2) \psi$.

The fermionic path integral can be evaluated by noting that the scalar field can be removed from the fermionic terms by a chiral rotation. Let

$$W(\theta) = \int [d\psi d\bar{\psi}] \exp \left(i \int d^2x \bar{\psi} i \mathcal{D} \psi \right)$$

then the change $\theta \rightarrow \theta - \alpha$, for α infinitesimal, accompanied by a change of variables $\psi \rightarrow (1 + i\alpha g \gamma_5/2)\psi$ leaves the lagrangian invariant. There is, however, a Jacobian which is evaluated using Fujikawa's technique¹¹⁴. The quantity to be evaluated is $\text{tr } \gamma_5 \exp [-\mathcal{D}^2/M^2]$ in the limit $M \rightarrow \infty$; a straightforward calculation gives

$$W(\theta - \alpha) = W(\theta) - \int \frac{d^2x}{4\pi} [g^2(\partial^\mu \theta)(\partial_\mu \alpha) + m^2(2\alpha g) \sin(2g\theta)], \quad (\text{C.2})$$

whence

$$W(\theta) = \int \frac{d^2x}{4\pi} [\tfrac{1}{2}g^2(\partial\theta)^2 - m^2 \cos(2g\theta)] \quad (\text{C.3})$$

The full effective lagrangian for θ when N heavy fermions are integrated out is

$$\mathcal{L}_{\text{eff}} = \tfrac{1}{2} \left(1 + \frac{g^2 N}{4\pi} \right) (\partial\theta)^2 - \frac{Nm^2}{4\pi} \cos(2g\theta). \quad (\text{C.4})$$

The quantity $g^2 N/(4\pi)$ represents the loop factor in this case.

Let

$$\lambda = \frac{2g}{\sqrt{1 + g^2 N/(4\pi)}}, \quad c = \sqrt{1 + \frac{4\pi}{g^2 N}}; \quad (\text{C.5})$$

then a field redefinition $\chi = (\pi + 2g\theta)/\lambda$ gives

$$\mathcal{L}_{\text{eff}} = \tfrac{1}{2}(\partial\chi)^2 + \frac{m^2 N}{4\pi} \cos(\lambda\chi) \quad (\text{C.6})$$

The mass of the linear excitations of this Sine-Gordon effective lagrangian (for a review see Ref. 62) equals $2m/c$ (where c is defined in (C.5)). The soliton mass equals mNc/π . By definition

$c \geq 1$, which implies that the soliton masses are always large, $O(m)$; the same will be true for all other “topological” objects in the model, such as breathers, etc. The linear excitations will be light only if $c \gg 1$ which is equivalent to $4\pi \gg g^2 N$.

If the number of fermion loops is so large as to offset the loop factor $g^2/(4\pi)$ there are no light excitations at all. If g is assumed to decrease with N , $g = G/\sqrt{N}$, then $c = \sqrt{1 + 4\pi/G^2}$ and the soliton masses are linear in N .

APPENDIX D

The proof that the equations of motion can be used to reduce the number of operators can be given in general⁶³ (see also Ref. 4). Here I will present a simple proof for a non-gauge theory, the above references should be consulted for the general case.

I will denote the fields by χ and the classical action by $S(\chi)$. Suppose now that we have two operators \mathcal{O} and \mathcal{O}' such that

$$\mathcal{O}' = \mathcal{O} + \int d^4x \mathcal{A} \frac{\delta S}{\delta \chi} \quad (\text{D.1})$$

for some local quantity \mathcal{A} depending on the χ . The effective action is

$$S_{\text{eff}} = S + \int d^4x (\eta \mathcal{O} + \eta' \mathcal{O}') + \dots; \quad (\text{D.2})$$

the dots indicate higher dimensional operators. Let $S' = S + \int d^4x (\eta + \eta') \mathcal{O}$, then

$$\begin{aligned} S_{\text{eff}} &= S'(\chi) + \eta' \int d^4x \mathcal{A} \frac{\delta S}{\delta \chi} + \dots \\ &= S'(\chi) + \eta' \int d^4x \mathcal{A} \frac{\delta S'}{\delta \chi} + \dots \\ &= S'(\chi + \eta' \mathcal{A}) + \dots \end{aligned} \quad (\text{D.3})$$

Thus, to the order we are working, the effects of \mathcal{O}' are to replace $\eta \rightarrow \eta + \eta'$ and $\chi \rightarrow \chi + \eta' \mathcal{A}$.

The natural next step is to change variables $\chi \rightarrow \phi = \chi + \eta' \mathcal{A}$. The Jacobian is one since \mathcal{A} is a local object and so $\delta \mathcal{A}(x)/\delta \chi(y)$ will be proportional to $\delta(x-y)$ or one of its derivatives. But in dimensional regularization all these quantities vanish when $x = y$, and so there is no contact term. In other regularization prescriptions the Jacobian will cancel some contact terms generated by S' .

To find the effects of the above change of variables I use an argument presented in Ref. 42. When considering Green's functions the replacement $\chi \rightarrow \phi$ generates many extra terms, especially if \mathcal{A} contains terms with several fields. But the contributions of the terms in \mathcal{A} with more than one field to any Green function do not have the physical particle propagator poles. Hence, when these terms are multiplied by the inverse propagators and the mass-shell condition is imposed, they vanish.

The only remaining terms are those where \mathcal{A} is linear in the fields χ . When these objects are close to the mass shell their effect is to multiply the $\eta' = 0$ contributions by a finite factor. This precise same factor will also appear in the propagator. Therefore when these terms are multiplied by inverse propagators and put on the mass-shell all the remaining effects of \mathcal{A} cancel.

From this discussion it follows that the only S-matrix effect of adding a redundant operator to \mathcal{L}_{eff} is to shift the couplings of the existing terms. This shift, as explained in section 5, can have a very important quantitative effect. If a consistent parametrization of Green functions is desired, then the full set of operators (equivalent or not) must be included in the effective lagrangian.

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